A GPU Algorithm for Greedy Graph Matching

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Outline

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- Matching has applications in
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- Our primary interest is graph coarsening, where we contract matched vertices to obtain a coarser version of the original graph.

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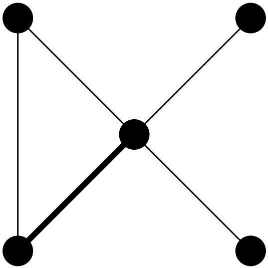
- All edges $e \in E$ are of the form $e = \{v, w\}$ for vertices $v, w \in V$.
- A matching is a collection $M \subseteq E$ of edges that are disjoint.
- We will view matchings as a map $\pi : V \to \mathbb{N}$ such that

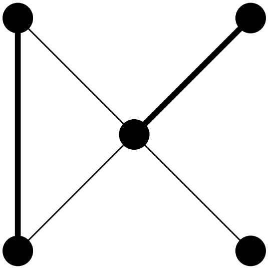
$$\pi(v) = \pi(w) \qquad \Longleftrightarrow \qquad \{v, w\} \in M.$$

Maximal Matching



- A matching is maximal if we cannot enlarge it further by adding another edge to it.

Maximum Matching



- A matching is maximum if it possesses the largest possible number of edges, compared to all other matchings.

Graph Matching

- If the edges are provided with weights $\omega : E \to \mathbb{R}_{>0}$, finding a matching M which maximises

$$\omega(M) = \sum_{e \in M} \omega(e),$$

is called weighted matching.

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is called weighted matching.
- Greedy matching provides us with maximal matchings, but not necessarily of maximum possible weight or maximum number of vertices/edges.

CPU matching

- We will now look at a serial greedy algorithm which generates a maximal matching.

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- In random order, vertices $v \in V$ select and match neighbours one-by-one.
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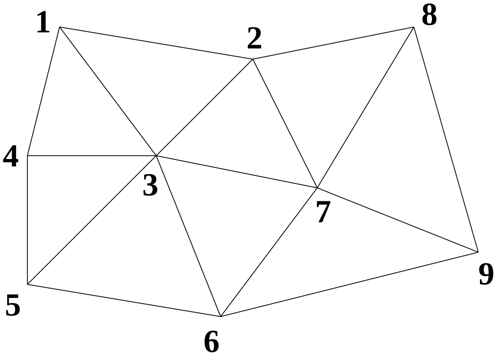
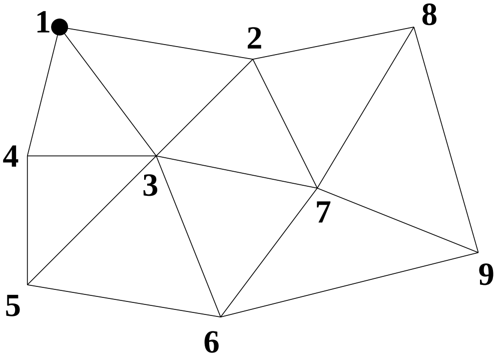
  - the neighbour w for which $\omega(\{v, w\})$ is maximal (weighted matching).

CPU matching



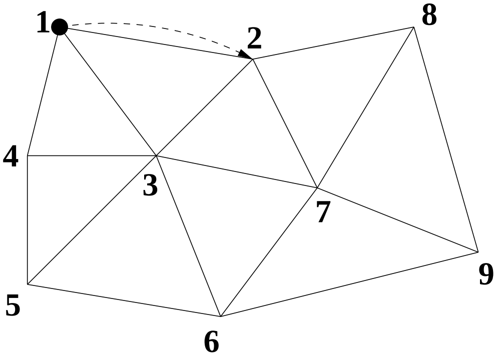
We will create a random matching for this graph.

CPU matching



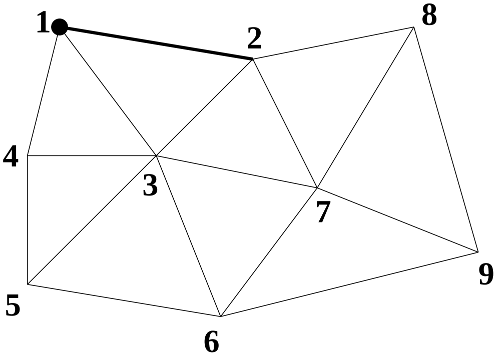
Consider the vertices one-by-one.

CPU matching



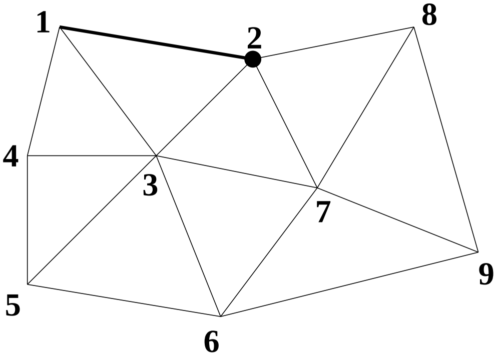
Select unmatched neighbour...

CPU matching



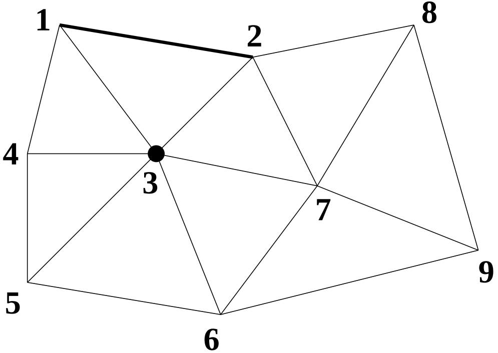
. . . and match.

CPU matching



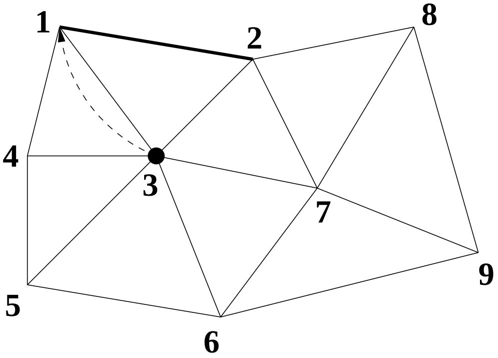
Skip matched vertices.

CPU matching



Skip already matched neighbours.

CPU matching



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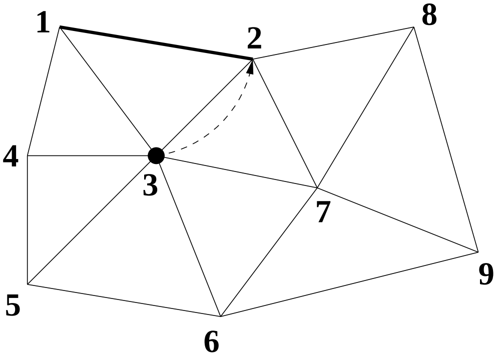

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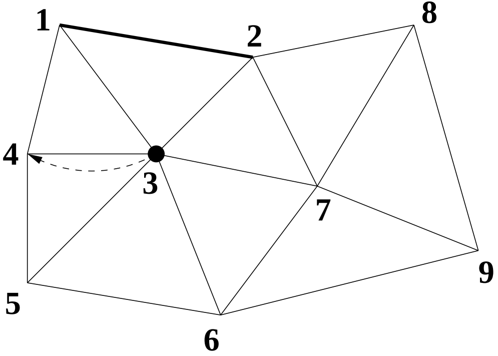
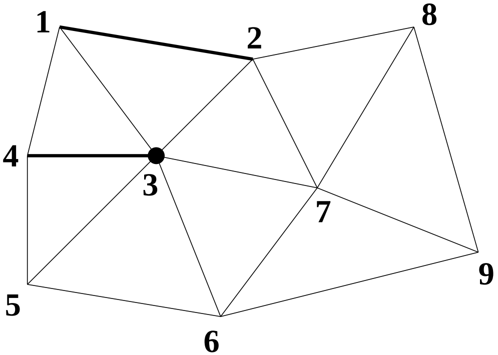


Keep matching until we have treated all vertices.

CPU matching



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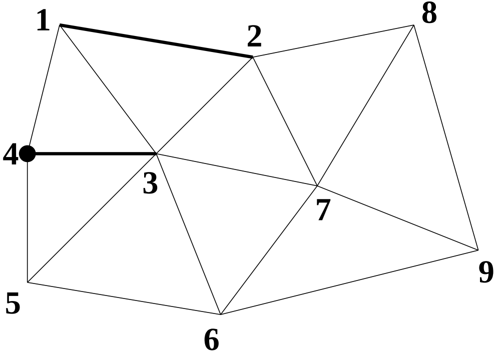

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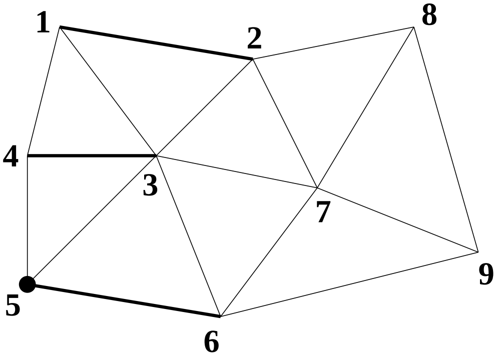

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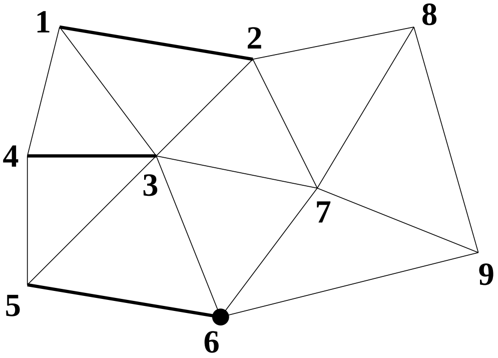

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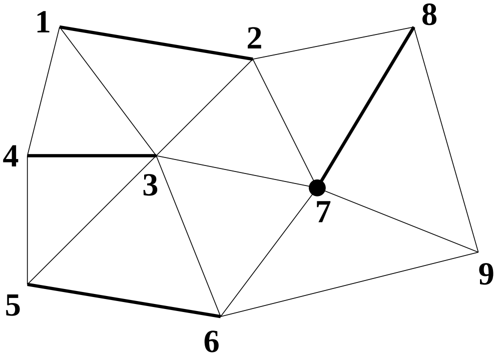

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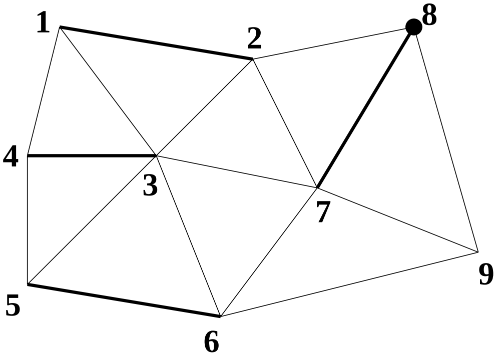

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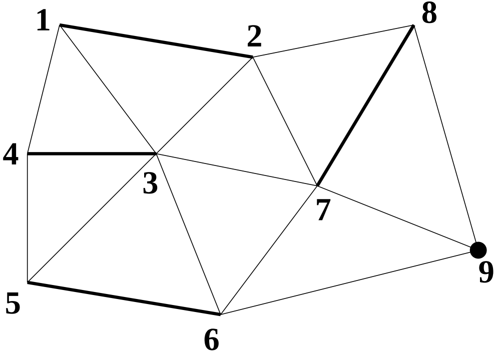

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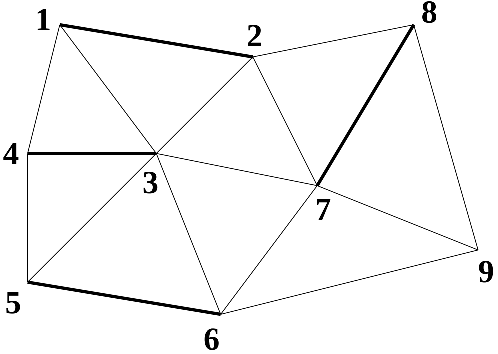

We have obtained a maximal matching (also maximum in this case).

Problematic parallelism

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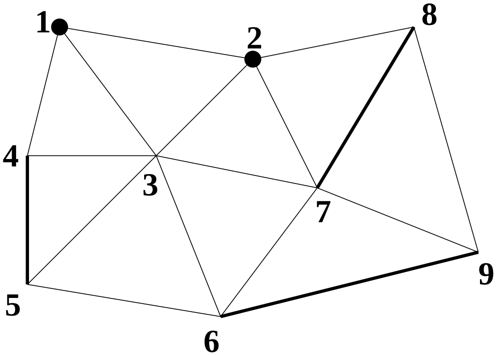
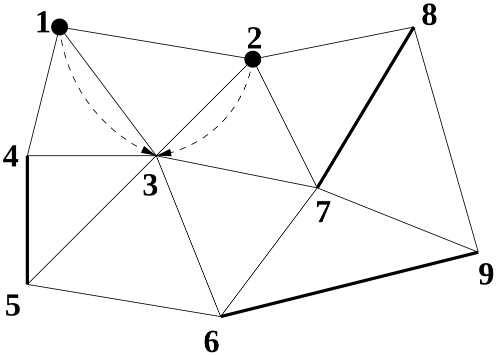
- Disjoint edges requirement leads to serialisation.

Problematic parallelism



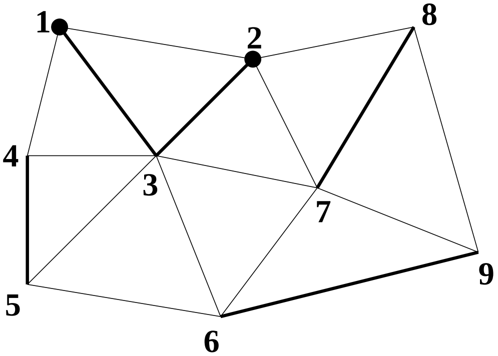
Suppose we match vertices simultaneously.

Problematic parallelism



Vertices find an unmatched neighbour. . .

Problematic parallelism



... but generate an invalid matching.

GPU matching

- To solve this we create two groups of vertices: **blue** and **red**.

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- **Blue** vertices propose.

GPU matching

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- **Blue** vertices propose.
- **Red** vertices respond.

GPU matching

- To solve this we create two groups of vertices: **blue** and **red**.
- **Blue** vertices propose.
- **Red** vertices respond.
- Proposals that were responded to are matched.

GPU implementation

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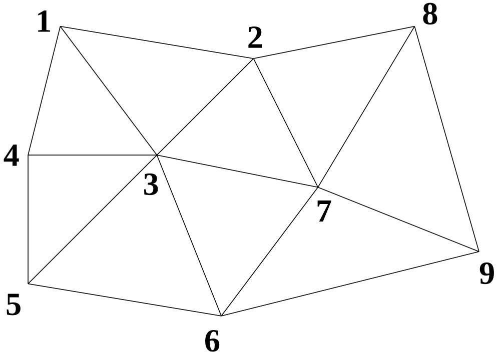
- We create one thread for each vertex in V.
- Each vertex $v \in V$ only updates
  - its colour/matching value $\pi(v)$;
  - and its proposal/response value $\sigma(v)$.
- Both π and σ are stored in 1D arrays in global memory.

GPU matching

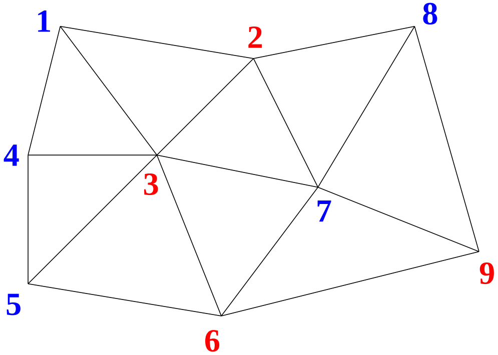
Colour
Propose
Respond
Match



	1	2	3	4	5	6	7	8	9
π	-	-	-	-	-	-	-	-	-
σ	-	-	-	-	-	-	-	-	-

GPU matching

Colour
Propose
Respond
Match



	1	2	3	4	5	6	7	8	9
π	**b**	**r**	**r**	**b**	**b**	**r**	**b**	**b**	**r**
σ	-	-	-	-	-	-	-	-	-

GPU matching

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Propose
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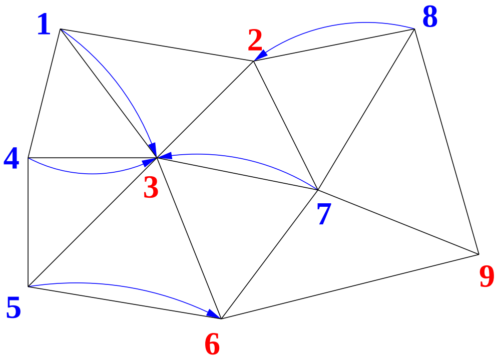
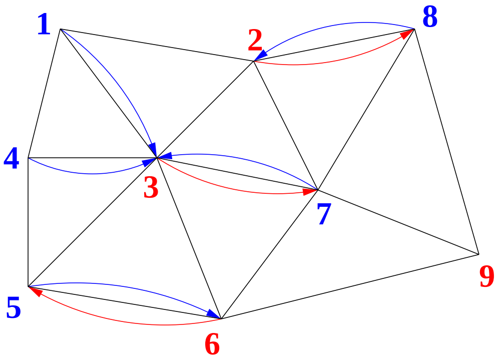


	1	2	3	4	5	6	7	8	9
π	**b**	**r**	**r**	**b**	**b**	**r**	**b**	**b**	**r**
σ	3	-	-	3	6	-	3	2	-

GPU matching

Colour
Propose
Respond
Match



	1	2	3	4	5	6	7	8	9
π	**b**	**r**	**r**	**b**	**b**	**r**	**b**	**b**	**r**
σ	3	8	7	3	6	5	3	2	-

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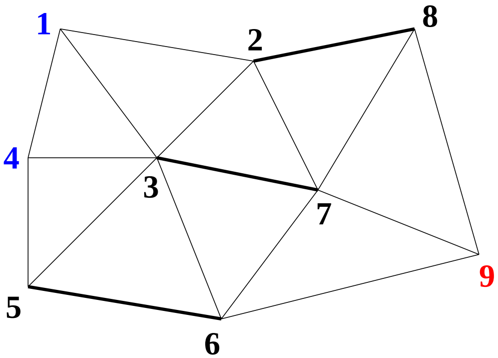

	1	2	3	4	5	6	7	8	9
π	**b**	2	3	**b**	5	5	3	2	**r**
σ	3	8	7	3	6	5	3	2	-

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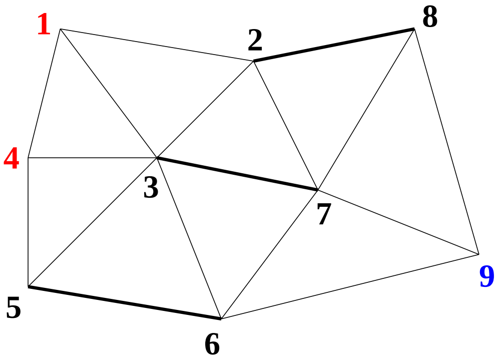
Match



	1	2	3	4	5	6	7	8	9
π	**r**	2	3	**r**	5	5	3	2	**b**
σ	3	8	7	3	6	5	3	2	-

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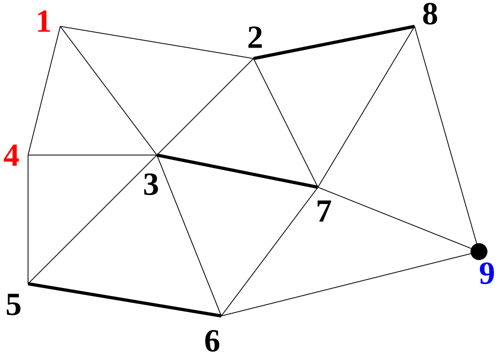
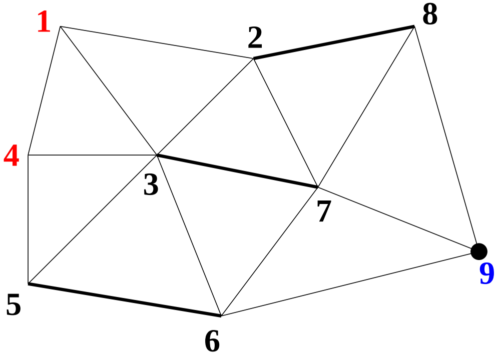


	1	2	3	4	5	6	7	8	9
π	**r**	2	3	**r**	5	5	3	2	**b**
σ	-	-	-	-	-	-	-	-	**d**

GPU matching

Colour
Propose
Respond
Match



	1	2	3	4	5	6	7	8	9
π	**r**	2	3	**r**	5	5	3	2	**b**
σ	-	-	-	-	-	-	-	-	**d**

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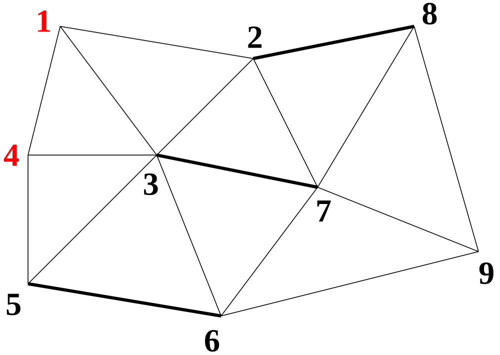

	1	2	3	4	5	6	7	8	9
π	**r**	2	3	**r**	5	5	3	2	**d**
σ	-	-	-	-	-	-	-	-	**d**

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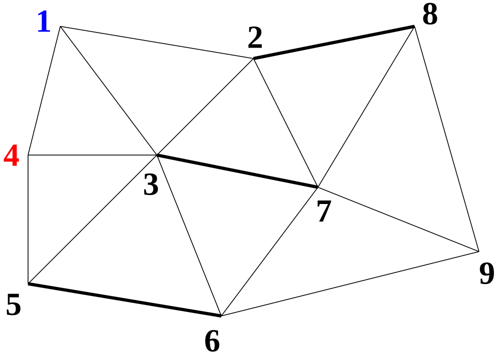
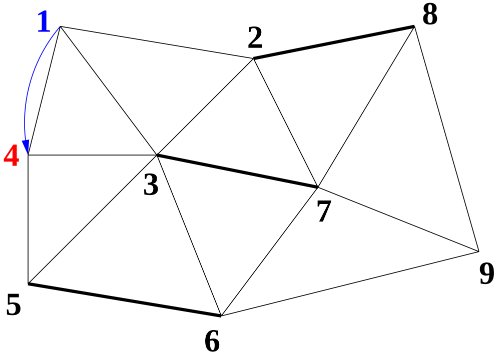


	1	2	3	4	5	6	7	8	9
π	**b**	2	3	**r**	5	5	3	2	**d**
σ	-	-	-	-	-	-	-	-	**d**

GPU matching



Colour
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Respond
Match

	1	2	3	4	5	6	7	8	9
π	**b**	2	3	**r**	5	5	3	2	**d**
σ	4	-	-	-	-	-	-	-	-

GPU matching

Colour
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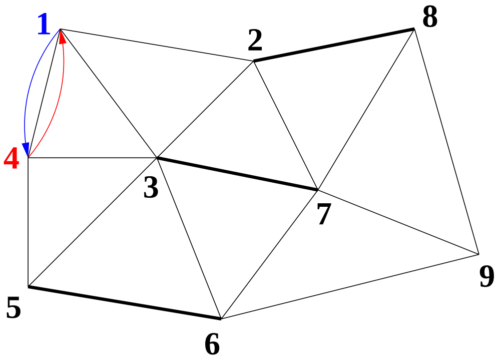

	1	2	3	4	5	6	7	8	9
π	**b**	2	3	**r**	5	5	3	2	**d**
σ	4	-	-	1	-	-	-	-	-

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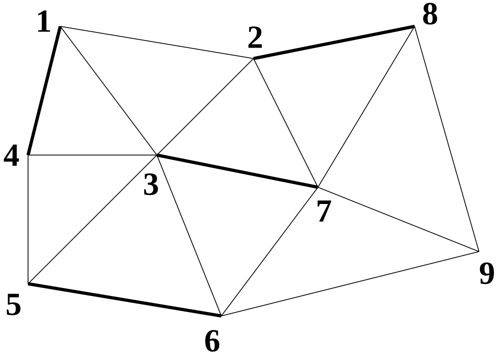
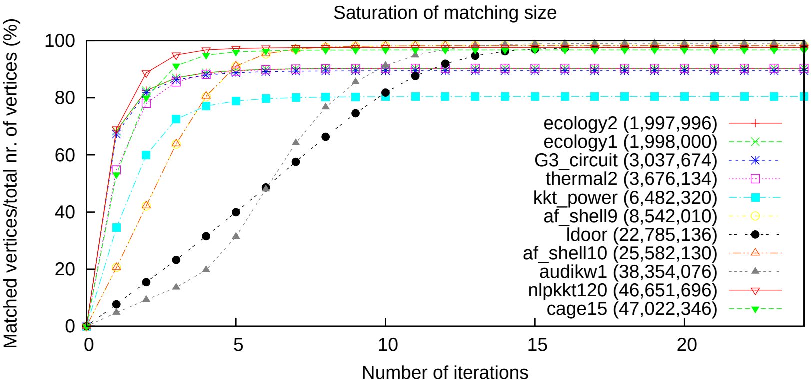


	1	2	3	4	5	6	7	8	9
π	1	2	3	1	5	5	3	2	**d**
σ	4	-	-	1	-	-	-	-	-

Matching saturation



Fraction of matched vertices as function of the number of iterations.

Colouring vertices

- To colour vertices $v \in V$, we use for a fixed $p \in [0, 1]$

$$\textbf{colour}(v) = \begin{cases} \textbf{blue} & \text{with probability } p, \\ \textbf{red} & \text{with probability } 1 - p. \end{cases} \tag{1}$$

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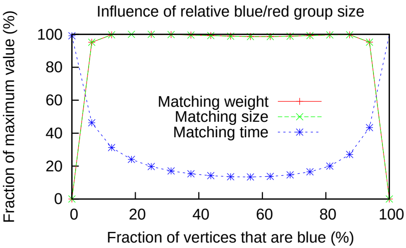
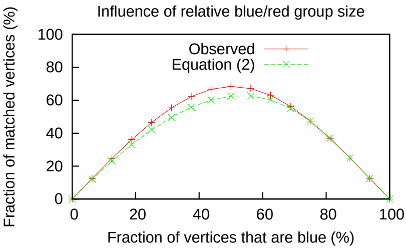
- How to choose p? Maximise the number of matched vertices.
- For a large random graphs, the expected fraction of matched vertices can be approximated by (independent of edge density)

$$2\,(1-p)\left(1 - e^{-\frac{p}{1-p}}\right). \tag{2}$$

Choosing p





Equation (2): we should choose $p \approx 0.53406$.

Results

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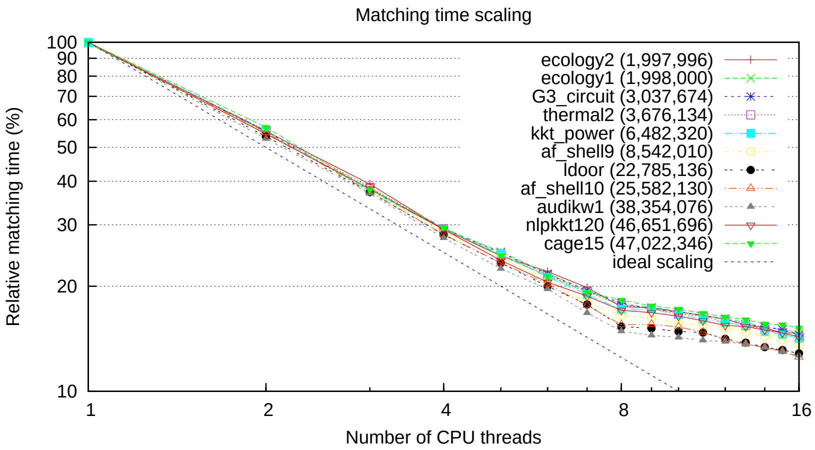
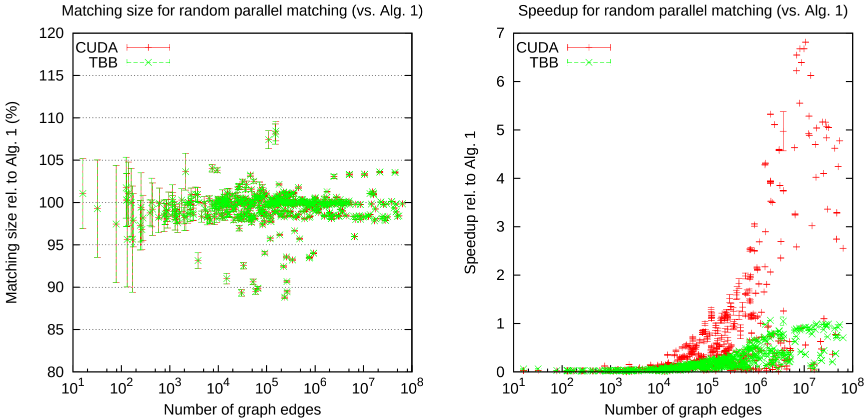
- Test set: ongoing 10th DIMACS challenge on graph partitioning and University of Florida Sparse Matrix Collection.
- Test hardware: dual quad-core Xeon E5620 and an NVIDIA Tesla C2050 (thanks: the Little Green Machine project).

Results (scaling)



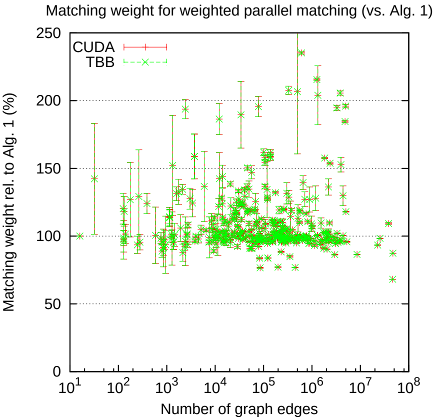
Scaling of TBB implementation (8 physical cores + hyperthreading).

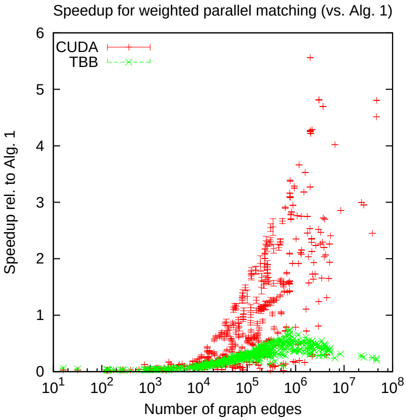
Results (vs. local random matching)



Matching size and speedup for parallel vs. serial local random matching.

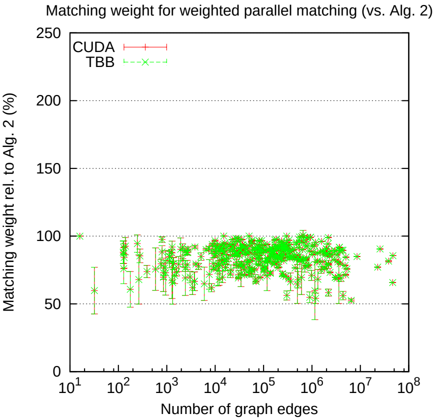
Results (vs. local weighted matching)

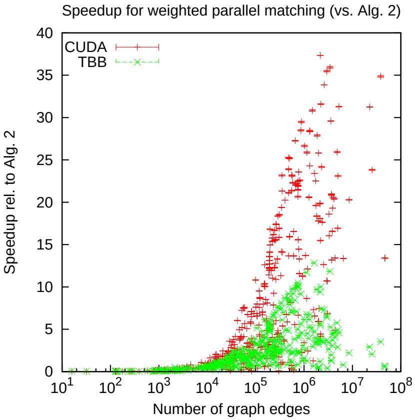




Matching weight and speedup for parallel vs. serial local weighted matching.

Results (vs. global weighted matching)





Matching weight and speedup for parallel local vs. serial global weighted matching.

Conclusion

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- Compared to a global greedy weighted matching algorithm quality is worse, but speedups up to 37 are achieved.
- We look forward to employ this algorithm in (hyper)graph coarsening.

Questions

$\exists$ any questions?

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- For a random graph with n vertices, we can approximate (independent of edge density)

$$\lim_{n\to\infty} \frac{2\,E(N(n))}{n} \approx 2\,(1-p)\,\left(1 - e^{-\frac{p}{1-p}}\right). \tag{3}$$

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$$\approx n\,(1 - p) \left(1 - \left(1 - \frac{p\,d}{1 + (1 - p)\,(d\,(n - 1) - 1)}\right)^{n-1}\right).$$

NVIDIA visual profiler