Differentiable manifolds – homework 3

Exercise 1. Read Section 4 ("Tangent vectors and differentials") from Warner's book.

Exercise 2. Check that scalar multiplication as defined in lectures

$$\lambda[\gamma] := [\varphi^{-1]}(\lambda\varphi \circ \gamma)]$$

does not depend on the choice of chart φ or of the path γ representing the class $[\gamma]$.

Exercise 3. Let $f : M \longrightarrow N$ be smooth and $X \in T_pM$. In lectures we defined

$$f_*X = [f \circ \gamma]$$

where $X = [\gamma]$. Show that this definition does not depend on the path γ chosen to represent X.

Exercise 4. Show that $f_* : T_pM \longrightarrow T_{f(p)}N$ is linear.

Exercise 5. Let $\varphi U \longrightarrow \mathbb{R}^n$ be a coordinate chart centered at p and let $x_i : U \longrightarrow \mathbb{R}$ be the composition of φ with the projection into the i^{th} coordinate in $\mathbb{R}^n$, $x_i = \pi_i \circ \varphi$. Show that $\{x_1, \cdots, x_n\}$ forms a basis for F_p/F_p^2 and hence T_pM is an n-dimensional vector space.

Exercise 6. Let $A : \mathbb{R}^n \longrightarrow \mathbb{R}^m$ be a linear map. Compute $A_*|_0 : T_0\mathbb{R}^n \longrightarrow T_0\mathbb{R}^m$.

Exercise 7. Let $A : \mathbb{R}^{n+1} \longrightarrow \mathbb{R}^{n+1}$ be an invertible linear map and define

$$\tilde{A} : S^n \longrightarrow S^n; \qquad \tilde{A}(v) = \frac{Av}{\|Av\|}.$$

Show that $\tilde{A}_*|_p$ is invertible for every $p \in S^n$.