

Sheet 7

Chapter 1.2, exercises: 6, 10, 11, 16, 19, 20.

Exercise 1. Let X be a connected graph, i.e., X is a 1 dimensional connected CW complex. A *tree* in X is a contractible subcomplex T . A *maximal tree* in X is a tree T which contains all the vertices of X . Show that if X is a connected finite graph, i.e, X has finitely many cells, then X has a maximal tree (or read Proposition 1 A1, page 84).

Exercise 2. Show that if X is a connected graph and $T \subset X$ is a maximal tree, then X/T is a wedge of circles (one circle for each edge in $X \setminus T$). Therefore conclude that $\pi_1(X)$ is a free group with one generator for each edge in $X \setminus T$.