Sheet 8

Exercise 1. Show that $T^2\#\mathbb{R}P^2 = 3\#\mathbb{R}P^2$.

Exercise 2. Solve exercise 16 from Chapter 1.2. Show that there is no way to add a boundary to the surface of the exercise so that it admits a triagulation with finitely many triangles (assume that adding a boundary does not change π_1).

Exercise 3.

1. Determine for which values of g, g', n and n' the surfaces $g\#T^2$ with n punctures and $g'\#T^2$ with n' punctures are homotopy equivalent.

2. Determine for which values of g, g', n and n' the surfaces $g\#\mathbb{R}P^2$ with n punctures and $g'\#\mathbb{R}P^2$ with n' punctures are homotopy equivalent.

3. Determine for which values of g, g', n and n' the surfaces $g\#T^2$ with n punctures and $g'\#\mathbb{R}P^2$ with n' punctures are homotopy equivalent.

Exercise 4. Given a Hausdorff topological space X, and a collection of compact subsets $\{K_i : i \in \mathbb{N}\}$ such that

- $K_i \subset K_{i+1}^\circ$, where K° denotes the interior of K and
- $\{K_i^\circ : i \in \mathbb{N}\}$ forms an open cover of X,

we say that an *end* of X is a collection $\{U_i : i \in \mathbb{N}\}$ of open sets such that U_i is a connected component of $X\backslash K_i$ and $U_{i+1} \subset U_i$.

Show that the number of ends of X does not depend on the particular collections $\{K_i : i \in \mathbb{N}\}$ satisfying the conditions above.

Exercise 5.

1. Determine for which values of g, g', n and n' the interior of the surfaces $g\#T^2$ with n punctures and $g'\#T^2$ with n' punctures are homeomorphic.

2. Determine for which values of g, g', n and n' the interior of the surfaces $g\#\mathbb{R}P^2$ with n punctures and $g'\#\mathbb{R}P^2$ with n' punctures are homeomorphic.

3. Determine for which values of g, g', n and n' the interior of the surfaces $g\#T^2$ with n punctures and $g'\#\mathbb{R}P^2$ with n' punctures are homeomorphic.