Riemann surfaces – Hand-in sheet 1

deadline: 22/Oct/10

Solve the following exercises from the book:
Chapter 3: 4, 5, 6 and 9;
Chapter 4: 3, 4, 5 and 6.

1) Let $\Sigma \subset \mathbb{C}P^2$ correspond to the zero set of the polynomial $p(z_0, z_1, z_2) = z_0 z_2^2 - z_1^3$ and consider the projection onto the first factors restricted to Σ:

$$\pi : \Sigma \backslash \{[0,0,1]\} \longrightarrow \mathbb{C}P^1 \qquad \pi(z_0, z_1, z_2) = [z_0, z_1].$$

- Check that π extends to a holomorphic map on Σ, i.e., it is well defined and holomorphic near the point $[0,0,1] \in \Sigma \subset \mathbb{C}P^2$;
- Resolve the singularity at $[1,0,0]$ to obtain a smooth Riemann surface $\overline{\Sigma} \subset \widetilde{\mathbb{C}P^2}$ in an appropriate blow-up of $\mathbb{C}P^2$
- Determine the degree of $\pi : \overline{\Sigma} \longrightarrow \mathbb{C}P^1$ and its branching points;
- Use the Riemann–Hurwitz formula

  $$\chi(\overline{\Sigma}) = deg(\pi)\chi(\mathbb{C}P^1) - B$$

  to determine the topological type of $\overline{\Sigma}$.
- Finally, also notice that the map $\varphi : \mathbb{C}P^1 \longrightarrow \mathbb{C}P^2$, $\varphi([z_0, z_1]) = [z_0^3, z_0 z_1^2, z_1^3]$ is a holomorphic map which maps $\mathbb{C}P^1$ bijectively onto Σ.