# Appendix

This appendix is part of the following contribution:

HILLEBRAND, A., VAN DEN AKKER, M., GERAERTS, R., AND HOOGEVEEN, H. 2016. Separating a walkable environment into layers. In *9th Int. ACM SIGGRAPH Conf. on Motion in Games*.

## A WEG Reductions

In [Hillebrand et al. 2016], several algorithms that can be used to reduce the size of the WEG are described. We have implemented these techniques and will test our algorithms on both the original WEG as well as the reduced WEG. For this reason, we will give a short description of the different graph reductions and how we have implemented them in this section. For the full details of these algorithms, we refer the reader to [Hillebrand et al. 2016].

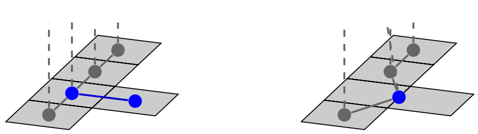
When performing the WEG reduction, we first create a set of candidate vertices on which a reduction operation might still be applied. This set of candidate vertices initially contains all the vertices in the WEG. We pick a candidate vertex from this set, and remove it. Next, we try each of the operations for the candidate vertex until we find one operation that can be applied. If none of the operations can be applied, we move on to the next candidate vertex, if any. But, if the operation was successful, we add the modified vertex as well as possibly influenced vertices to the set of candidate vertices. What vertices can be influenced by an operation is explained later.

These graph-reduction operations can be split into two categories: operations that are performed on vertices without overlaps (Appendix A.1) and operations that remove overlaps (Appendix A.2).

### A.1 Vertex reductions

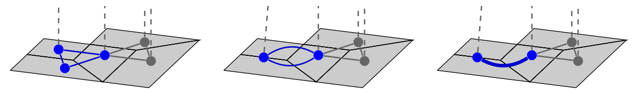
The methods in this category that we have implemented are `1-CONTRACT` and `2-CONTRACT` with `E-REDUCE` [Hillebrand et al. 2016]. `1-CONTRACT` removes a vertex $v$ that has an edge-degree of 1 and no overlaps. The only vertex that is possibly influenced by this operation is the single neighbour of $v$. An example of this operator and its application on a WEG can be found in Fig. 4.

The `2-CONTRACT` operator can be applied on a vertex with an edge-degree of 2 and no overlaps. This operation replaces the vertex $v$ and the two edges $e_1$ and $e_2$ with a single edge of weight $\min(w(e_1), w(e_2))$. When applying the `2-CONTRACT` operator, the two neighbours of $v$ are influenced and should therefore be added to the set of candidate vertices. In our implementation we combined the `2-CONTRACT` operator with the `E-REDUCE` operator. This operator can be applied to a vertex $v$ that has two edges $e_1$ and $e_2$ to a single vertex $w$. These two edges are replaced by a single edge of weight $w(e_1) + w(e_2)$. Combining these two operators is efficient, since the situation where `E-REDUCE` can be applied can only be the result of applying `2-CONTRACT`. Therefore, the detection of these cases can be done efficiently directly after applying the `2-CONTRACT` operator. A WEG on which we have applied these two operations is given in Fig. 5.



**(a)** **(b)**

**Figure 4:** *An example of the `1-CONTRACT` operation. The vertices and edges that will be modified are coloured blue. The dashed edges are overlap annotations. (a): A WEG before application of `1-CONTRACT`. (b): The WEG after application of `1-CONTRACT`. The remaining blue vertex now represents two polygons.*



**(a)** **(b)** **(c)**

**Figure 5:** *An example of `2-CONTRACT` combined with `E-REDUCE`. The vertices and edges that will be modified are coloured blue and dashed edges are overlap annotations. (a): A WEG before application of `2-CONTRACT`. (b): The WEG after application of `2-CONTRACT`. The leftmost vertex now represents two polygons. (c): The WEG after application of `E-REDUCE`.*

### A.2 Overlap removal

The operations that can reduce the number of overlaps that we have implemented are `T-CUT`, `STACK-REMOVE` and `d-REMOVE`. The `T-CUT` operator can be applied on two neighbouring vertices $v$ and $w$ that also overlap. The edge connecting $v$ and $w$ is added to the list of connections. Next, we also check if $v$ and $w$ have become disconnected. If $v$ and $w$ are in the same graph component, we are finished and add $v$ and $w$ to the set of candidate vertices. However, when $v$ and $w$ are disconnected, the overlap between $v$ and $w$ is also removed from the graph. Also, the vertices that need to be added to the set of candidate vertices are all the vertices from the two graph components that contain $v$ or $w$.

`STACK-REMOVE` can be applied on vertex $v$ that has edge-degree 2 and overlaps with a vertex $w$ that has also edge-degree 2. Furthermore, the neighbours of $v$ must overlap the neighbours of $w$. A situation like this can occur when there are two bridges that overlap. In this situation, the overlap between $v$ and $w$ can be discarded, since the separation of $v$ and $w$ is already guaranteed by the remainder of the environment. When this operation is applied for a vertex $v$ and removes the overlap $(v, w)$, both $v$ and $w$ need to be added to the set of candidate vertices.

The last operator, `d-REMOVE`, can be applied on any vertex $v$ that has overlaps. This operation enumerates all the simple paths of length $d$ that originate in $v$. For each simple path, we temporarily remove the vertices that are overlapped by vertices on the simple path. After doing this, a Breadth First Search (BFS) is started from the last vertex on the simple path. If we encounter a vertex $o$ that overlaps with $v$ during this BFS, we remember it. This process is repeated for all the enumerated simple paths. For overlaps that were not encountered during this process, there must be vertices on the simple paths of length $d$ that guarantee the separation. Therefore, we can safely remove the overlaps that were not encountered during this process from the WEG. The candidate set has to be updated if an overlap is removed. When this happens, $v$ and all vertices from which an overlap was removed have to be inserted.

## B Results

The results of the experiments are given in Table 3. It shows the performance of the three different algorithms for 13 environments for both the original and reduced WEG. Table 4 shows the result of the statistical analysis of the obtained results. For a select number of environments, an MLE is shown in Fig. 6.

**Table 3:** *The results of the experiments done on the various environments. Columns ending in 'sd' show the standard deviation of the previous column, using the same units. The columns with $|C|$ stand for the cumulative weight of the cut set, i.e. the number of connections. For some of the ILP experiments, no time was reported. This is because we stopped these runs after one hour. For these runs, the number of connections found thus far were reported.*

| | **Environment** | **local search** | | | | **HH** | | | | **ILP** | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | $\|C\|$ | $\|C\|$ sd | t (ms) | t sd | $\|C\|$ | $\|C\|$ sd | t (ms) | t sd | $\|C\|$ | $\|C\|$ sd | t (ms) | t sd |
| Original WEG | As_oilrig | 25.00 | 0.56 | $5.27e^4$ | $2.99e^3$ | 28.50 | 2.12 | $9.27e^1$ | $1.08e^1$ | — | — | — | — |
| | Halo | 4.00 | 0.00 | $6.21e^3$ | $3.47e^2$ | 4.35 | 0.67 | $3.15e^0$ | $3.66e^{-1}$ | 4.00 | 0.00 | — | — |
| | Cliffsides | 4.00 | 0.00 | $1.99e^4$ | $9.42e^2$ | 4.00 | 0.00 | $3.19e^1$ | $4.11e^0$ | — | — | — | — |
| | Hexagon | 29.20 | 0.70 | $1.09e^4$ | $6.51e^2$ | 31.00 | 1.45 | $1.56e^2$ | $8.96e^0$ | 57.50 | 8.02 | — | — |
| | Library | 8.25 | 0.44 | $6.41e^3$ | $1.71e^2$ | 9.00 | 0.00 | $6.30e^0$ | $5.71e^{-1}$ | — | — | — | — |
| | Tower | 38.35 | 1.14 | $1.93e^5$ | $2.06e^4$ | 40.40 | 1.60 | $8.85e^2$ | $8.65e^1$ | 65.00 | 0.00 | — | — |
| | Station 1 | 15.20 | 0.41 | $3.33e^3$ | $1.95e^2$ | 17.25 | 0.55 | $6.85e^0$ | $3.66e^{-1}$ | — | — | — | — |
| | Station 2 | 6.00 | 0.00 | $1.62e^3$ | $5.40e^1$ | 6.00 | 0.00 | $1.00e^0$ | $0.00e^0$ | 6.00 | 0.00 | $4.56e^6$ | $3.67e^3$ |
| | Parking lot | 8.00 | 0.00 | $1.40e^3$ | $4.61e^1$ | 8.00 | 0.00 | $1.00e^0$ | $0.00e^0$ | 8.00 | 0.00 | $2.38e^4$ | $4.56e^2$ |
| | City | 392.95 | 4.70 | $6.88e^5$ | $3.39e^4$ | 444.55 | 6.53 | $3.05e^4$ | $1.55e^3$ | 25773.25 | 359.29 | — | — |
| | Tower 10 | 41.60 | 1.14 | $1.53e^5$ | $1.21e^4$ | 64.60 | 4.13 | $8.77e^2$ | $8.26e^1$ | 551.25 | 26.27 | — | — |
| | Tower 20 | 40.30 | 0.73 | $1.88e^5$ | $1.17e^4$ | 95.00 | 4.87 | $1.04e^3$ | $1.86e^2$ | 870.00 | 48.87 | — | — |
| | Tower 40 | 41.00 | 0.92 | $1.28e^5$ | $1.17e^4$ | 87.00 | 6.36 | $7.06e^2$ | $7.49e^1$ | — | — | — | — |
| Reduced WEG | As_oilrig | 26.05 | 0.89 | $2.37e^4$ | $1.69e^3$ | 27.95 | 0.89 | $5.29e^1$ | $2.31e^0$ | — | — | — | — |
| | Halo | 4.00 | 0.00 | $2.65e^3$ | $1.34e^2$ | 5.00 | 0.00 | $2.00e^0$ | $0.00e^0$ | 4.00 | 0.00 | $4.39e^6$ | $4.89e^3$ |
| | Cliffsides | 4.00 | 0.00 | $1.01e^3$ | $4.99e^1$ | 4.00 | 0.00 | $6.05e^0$ | $2.24e^{-1}$ | — | — | — | — |
| | Hexagon | 29.60 | 1.14 | $2.07e^3$ | $1.50e^2$ | 30.65 | 0.49 | $3.69e^1$ | $4.89e^{-1}$ | 21.33 | 9.29 | — | — |
| | Library | 8.10 | 0.31 | $3.34e^3$ | $1.46e^2$ | 9.00 | 0.00 | $4.05e^0$ | $2.24e^{-1}$ | 8.00 | 0.00 | — | — |
| | Tower | 36.60 | 0.68 | $1.37e^5$ | $1.53e^4$ | 40.60 | 3.02 | $5.68e^2$ | $4.19e^1$ | 845.25 | 85.80 | — | — |
| | Station 1 | 16.25 | 0.72 | $1.25e^3$ | $8.81e^1$ | 19.00 | 0.00 | $2.05e^0$ | $2.24e^{-1}$ | — | — | — | — |
| | Station 2 | 6.00 | 0.00 | $5.91e^2$ | $1.50e^1$ | 6.00 | 0.00 | $0.00e^0$ | $0.00e^0$ | 6.00 | 0.00 | $9.00e^2$ | $2.02e^0$ |
| | Parking lot | 8.00 | 0.00 | $1.05e^3$ | $2.88e^1$ | 8.65 | 0.49 | $1.00e^0$ | $0.00e^0$ | 8.00 | 0.00 | $2.44e^5$ | $2.14e^3$ |
| | Tower 10 | 41.35 | 1.66 | $1.03e^5$ | $6.85e^3$ | 63.65 | 2.21 | $4.90e^2$ | $3.16e^1$ | 1232.75 | 47.80 | — | — |
| | Tower 20 | 39.55 | 0.83 | $1.03e^5$ | $7.79e^3$ | 100.30 | 4.51 | $5.94e^2$ | $7.38e^1$ | — | — | — | — |
| | Tower 40 | 40.90 | 0.97 | $9.12e^4$ | $6.21e^3$ | 95.55 | 7.05 | $4.98e^2$ | $3.96e^1$ | — | — | — | — |

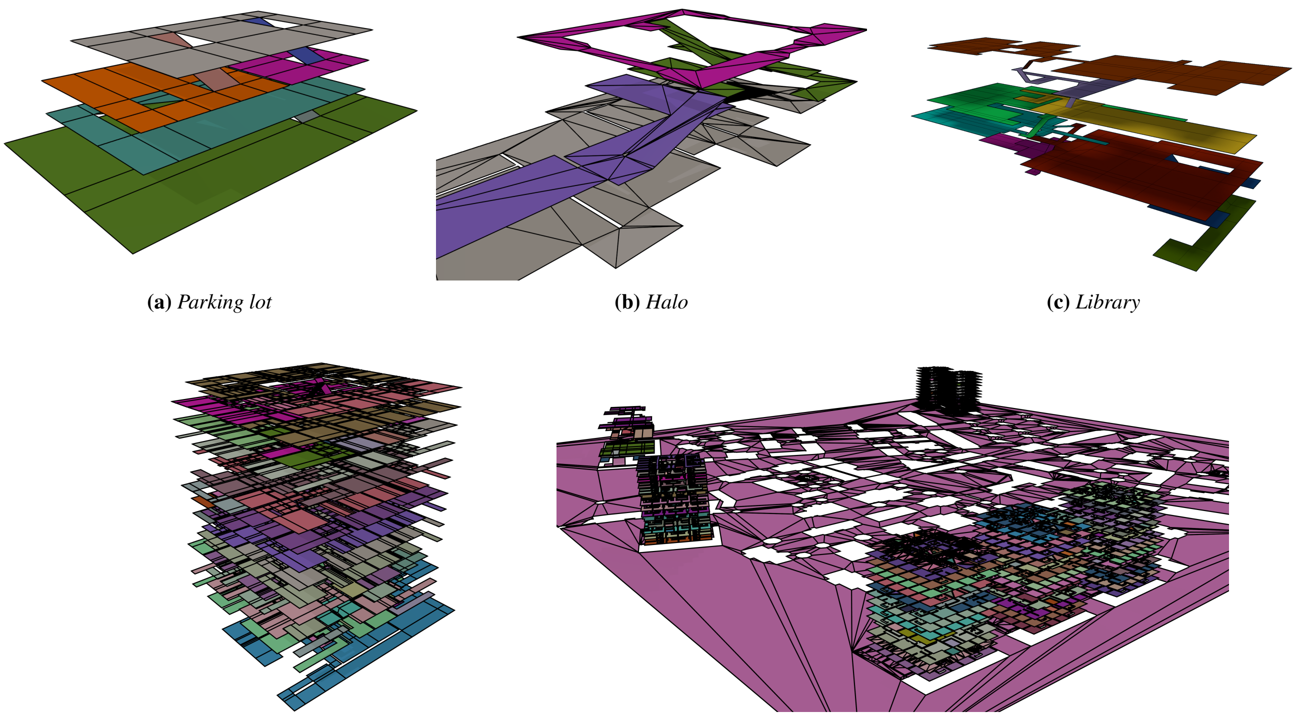
**Table 4:** *Table showing the confidence that there is a statistically significant difference between the algorithms 'Height Heuristic' (HH) and 'Local search' (LS). The suffix 'r' is added if this column concerns the experiments on the reduced WEG. The column $\alpha$ shows the significance level. In (a) the symbols △ and ▼ respectively mean that the algorithm was significantly faster or significantly slower. In (b) the symbols respectively mean that the algorithm found significantly better or significantly worse MLEs. The symbol — means that there was no significant difference. For example, we can say that HH is statistically significant faster than LS on a reduced graph for the Parking lot environment, but not for the City environment. For the same environment, performing the HH on the reduced graph is never significantly faster than performing local search, with or without first reducing the graph. To determine the significance levels, we used One-way ANOVA with the Tukey-Kramer method as post-hoc analysis.*

**(a)** *Running time*

| **Environ.** | $\alpha$ | HH | | | HHr | | LS |
|---|---|---|---|---|---|---|---|
| | | HHr | LS | LSr | LS | LSr | LSr |
| As_oilrig | 0.001 | — | △ | △ | △ | △ | ▼ |
| Halo | 0.001 | — | △ | △ | △ | △ | ▼ |
| Cliffsides | 0.001 | — | △ | △ | △ | △ | ▼ |
| Hexagon | 0.001 | — | △ | △ | △ | △ | ▼ |
| Library | 0.001 | — | △ | △ | △ | △ | ▼ |
| Tower | 0.001 | — | △ | △ | △ | △ | ▼ |
| Station 1 | 0.001 | — | △ | △ | △ | △ | ▼ |
| Station 2 | 0.001 | — | △ | △ | △ | △ | ▼ |
| Parking lot | 0.001 | — | △ | △ | △ | △ | ▼ |
| City | 0.001 | — | △ | — | — | — | — |
| Tower 10 | 0.001 | — | △ | △ | △ | △ | ▼ |
| Tower 20 | 0.001 | — | △ | △ | △ | △ | ▼ |
| Tower 40 | 0.001 | — | △ | △ | △ | △ | ▼ |

**(b)** *Number of connections*

| **Environ.** | $\alpha$ | HH | | | HHr | | LS |
|---|---|---|---|---|---|---|---|
| | | HHr | LS | LSr | LS | LSr | LSr |
| As_oilrig | 0.001 | — | ▼ | ▼ | ▼ | ▼ | — |
| Halo | 0.001 | △ | — | — | ▼ | ▼ | — |
| Cliffsides | — | — | — | — | — | — | — |
| Hexagon | 0.001 | — | ▼ | ▼ | ▼ | — | — |
| Library | 0.001 | — | ▼ | ▼ | ▼ | ▼ | — |
| Tower | 0.001 | — | — | ▼ | — | ▼ | — |
| Station 1 | 0.001 | △ | ▼ | ▼ | ▼ | ▼ | △ |
| Station 2 | — | — | — | — | — | — | — |
| Parking lot | 0.001 | △ | — | — | ▼ | ▼ | — |
| City | 0.001 | — | ▼ | — | — | — | — |
| Tower 10 | 0.001 | — | ▼ | ▼ | ▼ | ▼ | — |
| Tower 20 | 0.001 | △ | ▼ | ▼ | ▼ | ▼ | — |
| Tower 40 | 0.001 | △ | ▼ | ▼ | ▼ | ▼ | — |



**(a)** *Parking lot* **(b)** *Halo* **(c)** *Library*

**(d)** *Tower* **(e)** *City*

**Figure 6:** *Obtained MLEs for a select number of environments. These MLEs were obtained using the height heuristic.*