Performing multicut on walkable environments

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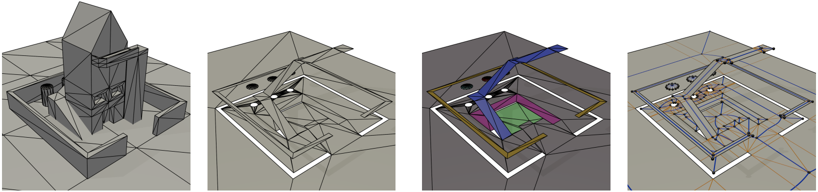
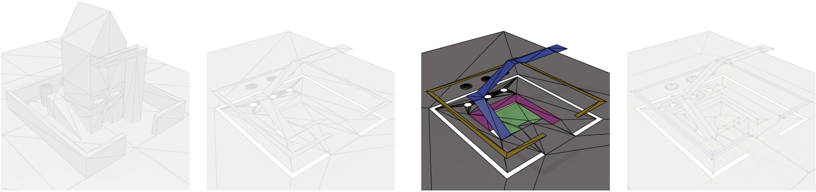
What?



1. Obtain a 3D-model of a building;
2. Filter and repair to obtain the **walkable environment**;
3. Obtain a **multi-layered environment**;
4. Do something useful (e.g. generate a **navigation mesh**).

What?



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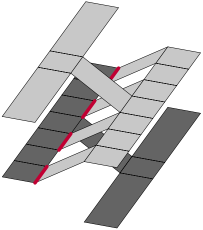
Multi-layered environment



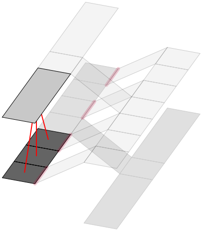
Definition (Multi-layered environment [1])

A multi-layered environment (MLE) is a set $\mathbf{L} = \{L_1, ..., L_l\}$ of layers and a set $\mathbf{C}$ of connections, such that:

- No two polygons P and Q in a layer L_i **overlap**;
- Iff polygons P and Q are **connected** and in different layers, the shared edge between P and Q is a **connection** in $\mathbf{C}$;
- Every polygon P is assigned to exactly one **layer**.

[1] van Toll, Cook IV, and Geraerts, "Navigation meshes for realistic multi-layered environments"

Multi-layered environment



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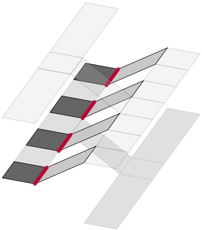

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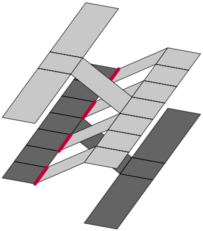

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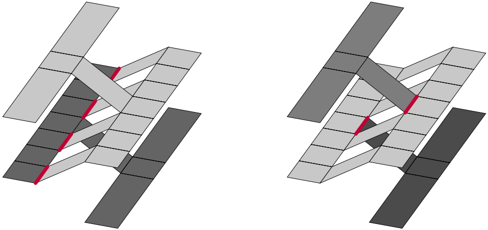
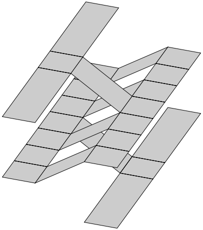
Minimally connected multi-layered environment



Definition (Minimally connected multi-layered environment)

A minimally connected multi-layered environment (MICLE) is a multi-layered environment where the number of connections is minimal.

Walkable environment graph

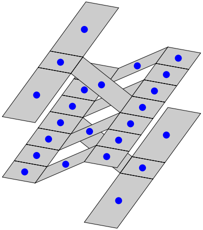


Definition (Walkable environment graph)

A walkable environment graph (WEG) is a graph representing a walkable environment with:

- A **vertex** for every polygon;
- An **edge** between every distinct pair of connected polygons;
- An **overlap** annotation between every distinct pair of overlapping polygons.

Walkable environment graph



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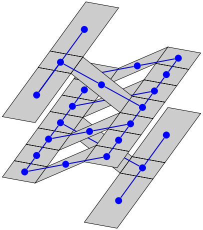

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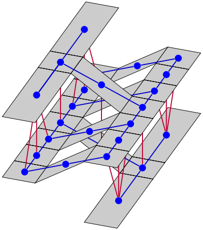

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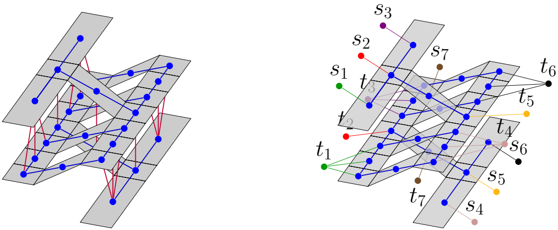
How hard can it be? (1/2)

Finding a MICLE can be formulated as a **MULTICUT** problem [1].

1. Create source s_x and sink t_x for a vertex v with overlaps;
2. Connect s_x to v;
3. Connect vertices overlapped by v to t_x and remove overlaps;
4. Repeat while there are still overlaps.



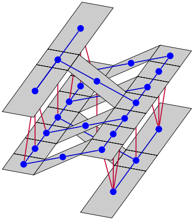
[1] Schrijver, *Combinatorial Optimization - Polyhedra And Efficiency*

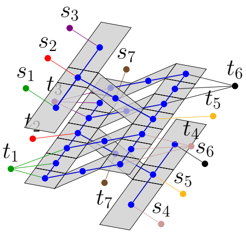
How hard can it be? (1/2)

Finding a MICLE can be formulated as a **MULTICUT** problem [1].

Unfortunately:

- MULTICUT is NP-Hard;
- MULTICUT is APX-Hard.





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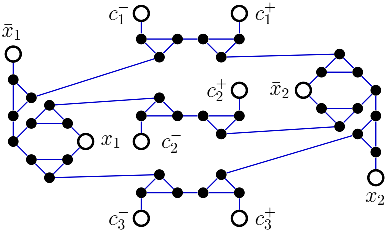
How hard can it be? (2/2)

In this paper we have proven that finding a MICLE is NP-Hard. This proof is based on earlier work of Dahlhaus et al. [1]



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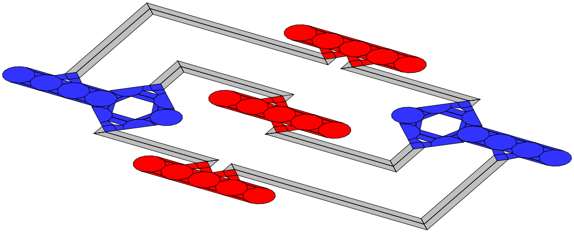

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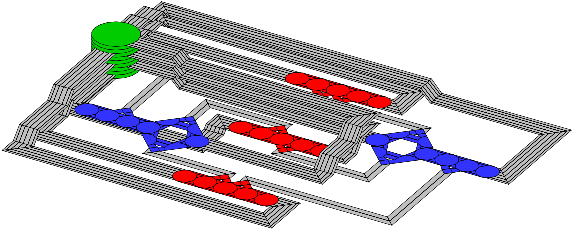

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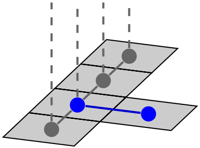
The general idea

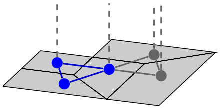
Observations:

- A WEG can contain a very large number of edges, vertices and overlaps.
- In some situations, vertices will always be part of the same layer.
- Some overlaps are redundant since their “constraint” is already enforced by other overlaps.

Try to efficiently **detect** these situations and remove them from a WEG. After each operation, keep track of possible new **candidate** vertices.

Vertex and edge reductions





1-CONTRACT

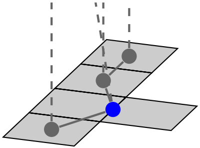
Detection: $O(1)$
Resolution: $O(p)$
Candidates: $O(1)$

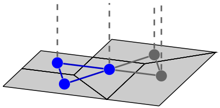
2-CONTRACT

Detection: $O(1)$
Resolution: $O(p)$
Candidates: $O(1)$

- p is the number of polygons a vertex represents.

Vertex and edge reductions





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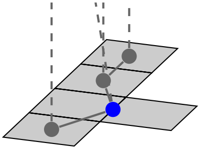

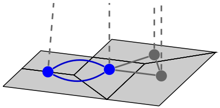

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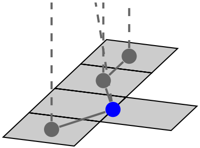

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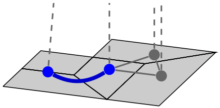

2-CONTRACT

Detection: $O(1)$
Resolution: $O(p)$
Candidates: $O(1)$

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Overlap removal

In general:

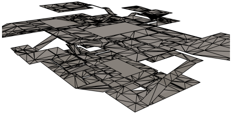
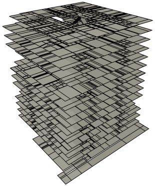
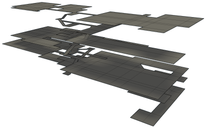
An overlap (v, w) can be removed if for every path connecting v and w there is a pair of overlapping vertices on that path.

! Very expensive to check

d-REMOVE

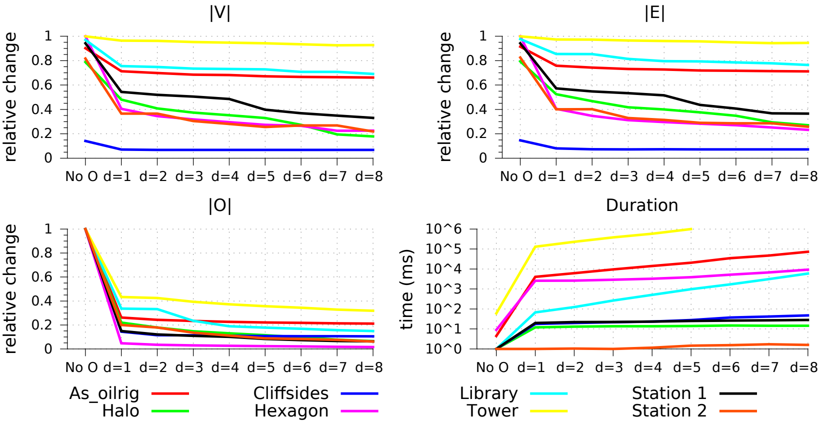
- Look at all sub-paths of length d originating in v
- For each sub-path register the overlaps
- Perform BFS from the end of each sub-path and search for w

Environments

Environment	**Type**	**Tri.**	$\vert V\vert$	$\vert E\vert$	$\vert O\vert$
As_oilrig	V	✓	2077	2399	10717
Halo	V	✓	179	184	346
Cliffsides	V	✓	748	764	162
Hexagon	V	✓	2368	2419	20207
Library	R	✗	298	420	775
Tower	R	✗	5932	8033	116983
Station 1	R	✓	206	209	1026
Station 2	R	✓	82	86	115

Results


|V|
relative change
1
0.8
0.6
0.4
0.2
0
No O d=1 d=2 d=3 d=4 d=5 d=6 d=7 d=8
|E|
relative change
1
0.8
0.6
0.4
0.2
0
No O d=1 d=2 d=3 d=4 d=5 d=6 d=7 d=8
|O|
relative change
1
0.8
0.6
0.4
0.2
0
No O d=1 d=2 d=3 d=4 d=5 d=6 d=7 d=8
Duration
time (ms)
10^6
10^5
10^4
10^3
10^2
10^1
10^0
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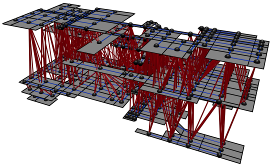
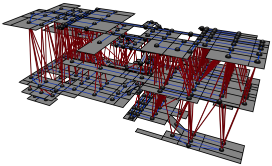
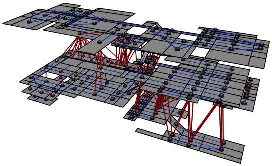


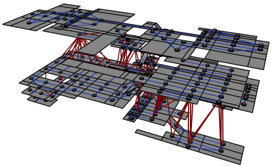
Original WEG



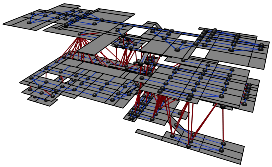
$d = 1$



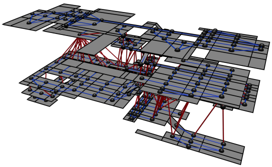
$d = 2$



$d = 3$



$d = 4$



$d = 5$

Conclusion

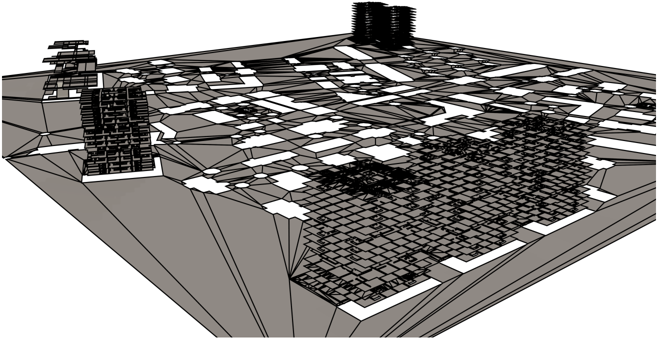
We have:

- Identified a common (sub-)problem: finding a MICLE;
- Proven this problem to be NP-Hard;
- Implemented and tested algorithms to shrink the problem size.

In the future:

- Work on the first step in the pipeline (extracting a WE);
- Extend 2D algorithms to multi-layered environments.

Thanks!



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