Practicum 2, 19 August 2009, 11:00-12:30 uur

For each planar system below, construct its phase portrait for $\alpha = 0$ and for small $\alpha < 0$ and $\alpha > 0$ using the `MATLAB` tool `pplane7` [1]. Identify the occuring bifurcation and try to support your conclusions by analytical arguments.

- **A system with a global bifurcation in which a local bifurcation is involved**

$$\begin{cases} \dot{x} &= x(1-x^2-y^2) - y(1+\alpha+x), \\ \dot{y} &= x(1+\alpha+x) + y(1-x^2-y^2). \end{cases} \tag{1}$$

  Hint: Introduce polar coordinates $x = r\cos\varphi,\ y = r\sin\varphi$.

- **Rayleigh's equation**

$$\ddot{x} + \dot{x}^3 - 2\alpha\dot{x} + x = 0. \tag{2}$$

  Hint: Introduce $y = \dot{x}$ and rewrite the equation as a system of two differential equations.

- **A system with a local bifurcation of a cycle**

$$\begin{cases} \dot{x} &= \left(\alpha - \frac{1}{4}\right)x - y + x(x^2+y^2) - x(x^4+2x^2y^2+y^4), \\ \dot{y} &= x + \left(\alpha - \frac{1}{4}\right)y + y(x^2+y^2) - y(x^4+2x^2y^2+y^4). \end{cases} \tag{3}$$

  Hint: Introduce polar coordinates $x = r\cos\varphi,\ y = r\sin\varphi$.

- **A system with a global bifurcation**

$$\begin{cases} \dot{x} &= 1 - x^2 - \alpha xy, \\ \dot{y} &= xy + \alpha(1-x^2). \end{cases} \tag{4}$$

- **A system with another global bifurcation**

$$\begin{cases} \dot{x} &= -x + 2y + x^2, \\ \dot{y} &= (2-\alpha)x - y - 3x^2 + \frac{3}{2}xy. \end{cases} \tag{5}$$

[1] When appropriate, plot orbits approaching a saddle point by selecting **Solutions → Plot stable and unstable orbits** in the `pplane7` Display Window and then pointing at the saddle with the mouse.