Numerical Bifurcation Analysis of Maps: Project

1 Model description

Consider the following discrete-time Hopfield model with delay for two coupled neurons

$$F(x,y): \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ y_1 \\ y_2 \\ y_3 \end{pmatrix} \mapsto \begin{pmatrix} ax_1 + T_{11}S(b_1x_2) + T_{12}S(b_2y_3) \\ x_1 \\ x_2 \\ ay_1 + T_{22}S(b_2y_2) + T_{21}S(b_1x_3) \\ y_1 \\ y_2 \end{pmatrix} \tag{1}$$

with passive decay a, gain factors $b_i > 0$ and coupling strength T_{ij} $(i,j = 1,2)$. The units activity x and y can be at steady state, excited (more positive) or suppressed (negative). The unit output is determined by a sigmoid function of the activity $S(u) = 1/(1 + exp(-u))$. Each unit has delayed self-excitation ($T_{ii} > 0$) or self-inhibition ($T_{ii} < 0$) and cross-communication that can be both excitatory ($T_{ij} > 0$) or inhibitory ($T_{ij} < 0$), with $i \neq j$.

2 A single neuron

Consider the case $T_{12} = 0, a = .25$ and study the (x_1, x_2)-subsystem with T_{11} and b_1 as free parameters. Here b_1 is positive and $T_{11} \in \mathbb{R}$. Give characteristic phase portraits for each region.

3 Interactions of neurons

Now we fix $a = .25$, $b_1 = b_2 = 4$ and $T_{11} = -T_{22} = -2$. Determine a numerical bifurcation diagram with free parameters (T_{12}, T_{21}) in the region $[-15, 15] \times [-15, 15]$.

1. Compute bifurcation curves corresponding to the first iterate.
2. Determine the pieces of the Neimark-Sacker bifurcations resulting in stable invariant curves. Use simulations to find phase-locking on the invariant curve. Use this as a starting point to compute several Arnol'd tongues.
3. Find bifurcation curves for the second iterate, in particular, compute the secondary Neimark-Sacker curve for the second iterate emerging from flip-NS points of the first iterate.
4. Determine the codim 2 bifurcations with only critical and stable multipliers. Describe the unfolding of these bifurcations based on the normal form coefficients.
5. Compute the secondary bifurcations near these bifurcations, excluding global ones. When bifurcations of tori are involved such as near double NS, flip-NS or fold-NS find an Arnol'd tongue that crosses this boundary.
6. Give characteristic phase portraits (in the (x_1, y_1)-plane around the codim 2 bifurcations of item 3.

Essay

Write an essay about your findings. Send it to `meijerhgeewi.utwente.nl` before April 24th.