Applied Bifurcation Theory: Practicum 2, 15 July 2019

Use the `MATLAB` bifurcation software `MatCont 7p1` to study the following systems and try to understand essential features of their phase portraits by relating your observations with the theory.

Ex.1 Rössler chaotic system

$$\begin{cases} \dot{x} &= -y - z \\ \dot{y} &= x + Ay \\ \dot{z} &= Bx - Cz + xz, \end{cases} \tag{1}$$

Fix $B = 0.4, C = 4.5$, and simulate the system for $A = 0.0, 0.2, 0.25, 0.3, 0.315$, and 0.36. Observe a supercritical Andronov-Hopf bifurcation and a transition to chaos via a cascade of period-doubling bifurcations.

Hint: Follow TUTORIAL I: USING MATCONT FOR NUMERICAL INTEGRATION OF ODES.

Ex.2 Torus is Langford system

$$\begin{cases} \dot{x} &= (\lambda - b)x - cy + xz + dx(1 - z^2), \\ \dot{y} &= cx + (\lambda - b)y + yz + dy(1 - z^2), \\ \dot{z} &= \lambda z - (x^2 + y^2 + z^2), \end{cases} \tag{2}$$

where $b = 3,\ c = \frac{1}{4}$ and $d = \frac{1}{5}$.

1. Simulate this system for $\lambda = 1.5, 1.9, 2.01$. Always start from $x_0 = y_0 = 0.1, z_0 = 1$.
2. Observe a supercritical Andronov-Hopf bifurcation followed by a Neimark-Sacker bifurcation that generates a stable invariant torus.
3. Could you support your observations by some other numerical and/or analytical arguments ? *Hint*: Introduce cylindrical coordinates (r, φ, z) so that $x = r\cos\varphi,\ y = r\sin\varphi$, and study the resulting (r, z)-system.

Ex.3 Arneodo system with a saddle-focus homoclinic bifurcation

$$\begin{cases} \dot{x} &= y, \\ \dot{y} &= z, \\ \dot{z} &= cx - by - z - x^2. \end{cases} \tag{3}$$

Fix $b = 0.5$ and simulate the system at $c = 0.960$ and $c = 0.965$. Which Shilnikov bifurcation happens between these two parameter values ? Approximate the bifurcation parameter value c_{HOM} numerically.

Ex.4 Blue-sky bifurcation in Gavrilov-Shilnikov system

$$\begin{cases} \dot{x} &= x(2 + \mu - b(x^2 + y^2)) + z^2 + y^2 + 2y, \\ \dot{y} &= -z^3 - (y + 1)(z^2 + y^2 + 2y) - 4x + \mu y, \\ \dot{z} &= z^2(y + 1) + x^2 - \varepsilon. \end{cases} \tag{4}$$

Fix $(b, \varepsilon) = (10, 0.02)$ and simulate the system at $\mu = 0.4$ and $\mu = 0.25$. Which bifurcation happens between these two parameter values ? Approximate the bifurcation parameter value μ_{BS} numerically.