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Measure and Integration Exercise 3, 2012-13

1. Let $(X, \mathcal{A}, \mu)$ be a measure space, and let $(X, \mathcal{A}^*, \overline{\mu})$ be its completion (see exercise 4.13, p.29).

   (a) Show that for any $f \in \mathcal{E}^+(\mathcal{A}^*)$, there exists a function $g \in \mathcal{E}^+(\mathcal{A})$ such that $g(x) \leq f(x)$ for all $x \in X$, and

   $$\overline{\mu}(\{x \in X : f(x) \neq g(x)\}) = 0.$$

   (b) Using Theorem 8.8, show that if $u \in \mathcal{M}^{\pm}_{\overline{\mathbb{R}}}(\mathcal{A}^*)$, then there exists $w \in \mathcal{M}^{\pm}_{\overline{\mathbb{R}}}(\mathcal{A})$ such that $w(x) \leq u(x)$ for all $x \in X$, and

   $$\overline{\mu}(\{x \in X : w(x) \neq u(x)\}) = 0.$$