Universiteit Utrecht
Mathematisch Instituut
Universiteit Utrecht
Boedapestlaan 6
3584 CD Utrecht

Measure and Integration Exercise 4, 2012-13

1. Consider the measure space $(\mathbb{R}, \mathcal{B}(\mathbb{R}), \lambda)$, where $\mathcal{B}(\mathbb{R})$ is the Borel σ-algebra over $\mathbb{R}$ and λ is Lebesgue measure on $\mathcal{B}(\mathbb{R})$. Let $f : \mathbb{R} \to \mathbb{R}$ be given by

$$f(x) = \begin{cases} 0 & \text{if } x < 0 \\ \\ 2^{-k} & \text{if } x \in [k, k+1),\ k \in \mathbb{Z},\ k \geq 0. \end{cases}$$

   (a) Show that f is measurable, i.e. $f^{-1}(B) \in \mathcal{B}(\mathbb{R})$ for all $B \in \mathcal{B}(\mathbb{R})$.

   (b) Determine the values of $\lambda(\{f > 1\})$, $\lambda(\{f < 1\}$ and $\lambda(\{1/4 \leq f < 1\}$.

   (c) Determine the value of $\int f \, d\lambda$.