Measure and Integration Hand in Exercises 6 2012-13

(1) Let $(X, \mathcal{A}, \mu)$ be a probability space (i.e. $\mu(X) = 1$).
   (a) Suppose $1 \leq p < r$, and $f_n, f \in \mathcal{L}^r(\mu)$ satisfy $\lim_{n\to\infty} ||f_n - f||_r = 0$. Show that $\lim_{n\to\infty} ||f_n - f||_p = 0$.
   (b) Assume $p, q > 1$ satisfy $1/p + 1/q = 1$. Suppose $f_n, f \in \mathcal{L}^p(\mu)$, and $g_n, g \in \mathcal{L}^q(\mu)$ satisfy
$$\lim_{n\to\infty} ||f_n - f||_p = \lim_{n\to\infty} ||g_n - g||_q = 0.$$
   Show that $\lim_{n\to\infty} ||f_n g_n - fg||_1 = 0$.