Universiteit Utrecht Universiteit Utrecht Budapestlaan 6

Mathematisch Instituut 3584 CD Utrecht

Measure and Integration 2012-13-extra exercises Chapter 13

1. (**Extra exercise 1**) Let $(X, \mathcal{A})$ and $(Y, \mathcal{B})$ be measure space, and let $(X \times Y, \mathcal{A} \otimes \mathcal{B})$ be the corresponding product measurable space, where $\mathcal{A} \otimes \mathcal{B} = \sigma(\mathcal{A} \times \mathcal{B})$.

   (a) Show that for all $E \in \mathcal{A} \otimes \mathcal{B}$, and for all $x_0 \in X$ and all $y_0 \in Y$, one has $E_{x_0} = \{y \in Y : (x_0, y) \in E\} \in \mathcal{B}$ and $E_{y_0} = \{x \in X : (x, y_0) \in E\} \in \mathcal{A}$.

   (b) Let $f : X \times Y \to \overline{\mathbb{R}}$ be $\mathcal{A} \otimes \mathcal{B}/\mathcal{B}(\overline{\mathbb{R}})$ measurable. Show that for all $x_0 \in X$ and all $y_0 \in Y$, the functions $f_{x_0} : Y \to \overline{\mathbb{R}}$ and $f_{y_0} : X \to \overline{\mathbb{R}}$ given by $f_{x_0}(y) = f(x_0, y)$ and $f_{y_0}(x) = f(x, y_0)$ are $\mathcal{A}/\mathcal{B}(\overline{\mathbb{R}})$ respectively $\mathcal{B}/\mathcal{B}(\overline{\mathbb{R}})$ measurable.

2. (**Extra exercise 2**) Suppose $(X, \mathcal{A}, \mu)$ and $(Y, \mathcal{B}, \nu)$ are σ-finite measure spaces. Let $f : X \to [0, \infty)$, $g : Y \to [0, \infty)$ be $\mathcal{A}/\mathcal{B}(\mathbb{R})$ respectively $\mathcal{B}/\mathcal{B}(\mathbb{R})$ measurable functions. Define $h : X \times Y \to [0, \infty)$ by $h(x, y) = f(x)g(y)$. Show that h is $\mathcal{A} \otimes \mathcal{B}/\mathcal{B}(\mathbb{R})$ measurable.