



Extra Opgaven Inleiding Financiele Wiskunde, 2011/12

1. Consider a 2-period binomial model with $S_0 = 10$, $u = 1.25$, $d = 0.75$, and $r = 0.2$. Suppose the real probability measure P satisfies $P(H) = p = 0.6 = 1 - P(T)$.

- (a) Consider an option with payoff $V_2 = (\max(S_1, S_2) - 11)^+$. Determine the price V_n at time $n = 0, 1$.
- (b) Suppose $\omega_1\omega_2 = HH$, find the values of the portfolio process $\Delta_0, \Delta_1(H)$ so that the corresponding wealth process satisfies $X_2(HH) = V_2(HH)$.
- (c) Determine explicitly the state price density process

$$\frac{Z_0}{(1+r)^0}, \frac{Z_1}{(1+r)}, \frac{Z_2}{(1+r)^2},$$

where

$$Z_2(\omega_1\omega_2) = Z(\omega_1\omega_2) = \frac{\tilde{P}(\omega_1\omega_2)}{P(\omega_1\omega_2)}$$

with \tilde{P} the risk neutral probability measure, and $Z_i = E_i(Z)$, $i = 0, 1, 2$.

2. Consider the N -period binomial model. Consider the random variables X_1, \dots, X_N on (Ω, P) defined by

$$X_i(\omega_1 \dots \omega_N) = \begin{cases} 2, & \text{if } \omega_i = H, \\ 0, & \text{if } \omega_i = T. \end{cases}$$

- (a) Assume $P(H) = 1/2 = P(T)$. Let $Z_0 = 1$, and $Z_n = X_1 \dots X_n$, $n = 1, 2, \dots, N$. Prove that the process Z_0, Z_1, \dots, Z_N is a martingale w.r.t. P .
- (b) Suppose $P(H) = 1/4 = 1 - P(T)$. Show that the process Z_0, Z_1, \dots, Z_N in part (a) is now a supermartingale w.r.t. P , while the process $Z_0^2, Z_1^2, \dots, Z_N^2$ is a martingale w.r.t. P .

3. Consider the N -period binomial model. Suppose Y is a random variable on Ω . Define a process

$$Y_0, Y_1, \dots, Y_N$$

by $Y_n = \tilde{E}_n(Y)$. Let

$$Z_0, Z_1, \dots, Z_N$$

be the Radon-Nikodym derivative process of \tilde{P} w.r.t. P , so $Z_n = E(Z)$, with Z the Radon-Nikodym derivative of \tilde{P} w.r.t. P . Show that the process

$$Z_0 Y_0, Z_1 Y_1, \dots, Y_N Z_N$$

is a martingale w.r.t. P . (Hint: use Lemma 3.2.6)