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Extra Opgaven Inleiding Financiele Wiskunde, 2011-12

1. Consider the binomial model with $u = 2^1$, $d = 2^{-1}$, and $r = 1/4$, and consider a perpetual American put option with $S_0 = 10$ and $K = 12$. Suppose that Alice and Bob each buy such an option

   (a) Suppose that Alice uses the strategy of exercising the first time the price reaches 5 euros. What should then the price be at time 0?

   (b) Suppose that Bob uses the strategy of exercising the first time the price reaches 2.5 euros. What should then the price be at time 0?

   (c) What is the probability that the price reaches 20 euros for the first time at time $n = 5$?

2. Let $M_0, M_1, \cdots$ be the symmetric random walk. Define for $a \in Z$, $M_n^a = a + M_n$. The process $M_0^a, M_1^a, \cdots$ is called the symmetric random walk starting in a. Let $b \in Z$ be such that $n + b - a$ is even.

   (a) Let $N_n(a, b)$ be the number of paths of length n starting in a and ending in b. Show that $N_n(a, b) = \binom{n}{\frac{1}{2}(n+b-a)}$. Conclude that

   $$P(M_n^a = b) = \binom{n}{\frac{1}{2}(n+b-a)} \frac{1}{2^n}.$$

   (b) Let $N_n^0(a, b)$ be the number of paths of length n starting in a and ending in b that cross the x-axis at least once. Use the reflection principle to prove that if $a, b > 0$, then $N_n^0(a, b) = N_n(-a, b)$.

   (c) Let $b > 0$, using part (b) show that the number of paths of length n starting in 0 which does not cross the x-axis (except at the starting point) equals $\frac{b}{n} N_n(0, b)$.

   (d) Use part (c) to prove that if $b > 0$, then

   $$P(M_n = b, \min_{1 \le k \le n-1} M_k > 0) = \frac{b}{n} P(M_n = b).$$