Solutions Book Chapter 5, SCI 113 Spring 2008

(1) **Exercise 5.1 (b)** $(1+x)^{1/2} \approx 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16}$. If we use a three term polynomial, then to get a two decimal precision we need $\frac{x^3}{16} < .005$ leading to $-0.43 < x < 0.43$. **(c)** $(1+x)^{-1/3} \approx 1 - \frac{x}{3} + \frac{2x^2}{9} - \frac{14x^3}{81}$. With three term polynomial, we get a two decimal precision if $\frac{14x^3}{81} < 0.005$, leading to $-0.306979 < x < 0.306979$. **(f)** $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4}$. With three term polynomial, we get a two decimal precision if $\frac{x^4}{4} < 0.005$, leading to $-0.37606 < x < 0.37606$. **(g)** $(1+x^2)^{1/2} \approx 1 + \frac{x^2}{2} - \frac{x^4}{8} + \frac{x^6}{16}$. If we use a three term polynomial, then to get a two decimal precision we need $\frac{x^6}{16} < .005$ leading to $-0.6564 < x < 0.6564$.

(2) **Exercise 5.2 (c)** If we calculate the successive derivatives, then we see that

$$f^{(n)}(x) = \begin{cases} \cos x & \text{if } n = 4m \\ -\sin x & \text{if } n = 4m+1 \\ -\cos x & \text{if } n = 4m+2 \\ \sin x & \text{if } n = 4m+3 \end{cases}$$

Evaluating the derivatives at 0, we see that $f^{(n)}(0) = 0$ if n is odd. For n even we have that

$$f^{(n)}(x) = \begin{cases} 1 & \text{if } n = 4m = 2(2m) \\ -1 & \text{if } n = 4m+2 = 2(2m+1) \end{cases}$$

as required.

(d) If we calculate the successive derivatives, then we see that

$$f^{(n)}(x) = \alpha(\alpha-1)(\alpha-2)\cdots(\alpha-n+1)(1+x)^{\alpha-n}.$$

Evaluating at 0, we get $f^{(n)}(0) = \alpha(\alpha-1)(\alpha-2)\cdots(\alpha-n+1)$ as required.

(3) **Exercise 5.4 (e)** $1-x$.

(4) **Exercise 5.10 (a)**

$$\ln x = \ln(1+(x-1)) = (x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} - \frac{(x-1)^4}{4} \cdots .$$

This formula is valid if $-1 < x-1 \leq 1$, i.e. $0 < x \leq 2$.

(b) First note that

$$\cos x = \cos(\frac{\pi}{2} + (x - \frac{\pi}{2})) = \cos\frac{\pi}{2}\cos(x-\frac{\pi}{2}) - \sin\frac{\pi}{2}\sin(x-\frac{\pi}{2}) = -\sin(x-\frac{\pi}{2}).$$

Thus,

$$\cos x = -\sin(x - \frac{\pi}{2}) = -(x - \frac{\pi}{2}) + \frac{(x - \frac{\pi}{2})^3}{3!} - \frac{(x - \frac{\pi}{2})^5}{5!} + \cdots .$$