Solutions Book Chapter 28, SCI 113 Spring 2008

(1) **Exercise 28.8 (a)** $\frac{\partial f}{\partial x} = 2x + 3y - 1$, $\frac{\partial f}{\partial y} = 4y + 3x$, $\frac{\partial^2 f}{\partial x^2} = 2$, $\frac{\partial^2 f}{\partial y^2} = 4$ $\frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y} = 3$, **(d)** $\frac{\partial f}{\partial x} = \frac{-y}{x^2}$, $\frac{\partial f}{\partial y} = \frac{1}{x}$, $\frac{\partial^2 f}{\partial x^2} = \frac{2y}{x^3}$, $\frac{\partial^2 f}{\partial y^2} = 0$, $\frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y} = \frac{-1}{x^2}$. **(h)** $\frac{\partial f}{\partial x} = 12(3x-4y)^3$, $\frac{\partial f}{\partial y} = -16(3x-4y)^3$, $\frac{\partial^2 f}{\partial x^2} = 108(3x-4y)^2$, $\frac{\partial^2 f}{\partial y^2} = 192(3x-4y)^2$, $\frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y} = -144(3x-4y)^2$.

(2) **Exercise 28.9** $\frac{\partial^2 f}{\partial x^2} = \frac{1}{r^2} - \frac{2x^2}{r^4}$, and $\frac{\partial^2 f}{\partial y^2} = \frac{1}{r^2} - \frac{2y^2}{r^4}$. Since $r^2 = x^2 + y^2$, we get $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$.

(3) **Exercise 28.10 (b)** $\frac{\partial f}{\partial x}(2,2,4) = 2 = \frac{\partial f}{\partial y}(2,2,4)$. Hence, equation of tangent plane is $(z-4) = 2(x-2) + 2(y-2)$. Normal vector is (2,2,-1) (or $2\mathbf{i}+2\mathbf{j}-\mathbf{k}$), **(d)** $\frac{\partial f}{\partial x}(3,4,2) = \frac{-9}{2}$, $\frac{\partial f}{\partial y}(3,4,2) = -2$, tangent plane: $(z-2) = \frac{-9}{2}(x-3) - 2(y-4)$, normal vector is $(\frac{-9}{2}, -2, -1)$. **(f)** $\frac{\partial f}{\partial x}(0,0,1) = 0$, $\frac{\partial f}{\partial y}(0,0,1) = 0$, tangent plane: $z = 1$ (parallel to the xy-plane), normal vector $(0,0,-1)$.

(4) **Exercise 28.11** We first look at the surface $z = x^2 + y^2$: $\frac{\partial f}{\partial x}(1,1,2) = 2$, $\frac{\partial f}{\partial y}(1,1,2) = 2$, so $\mathbf{n_1} = (2,2,-1)$. Now we look at the surface $z = x - y + 2$: $\frac{\partial f}{\partial x}(1,1,2) = 1$, $\frac{\partial f}{\partial y}(1,1,2) = -1$, so $\mathbf{n_2} = (1,-1,-1)$. Let θ be the angle between $\mathbf{n_1}$ and $\mathbf{n_2}$, then

$$\cos\theta = \frac{\mathbf{n_1} \cdot \mathbf{n_2}}{|\mathbf{n_1}|\,|\mathbf{n_1}|} = \frac{1}{3\sqrt{3}}.$$

Hence $\theta = 78.9°$ (or $101.1°$).

(5) **Exercise 28.12 (b)** minimum at $(1,-1)$, **(c)** saddle points at $(1,1)$ and $(-1,-1)$, minimum at $(1,-1)$, and maximum at $(-1,1)$, **(k)** a saddle point at $(0,0)$.