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Measure and Integration Exercises 1

1. Let J be a rectangle in $\mathbb{R}^n$, $c, d \in \mathbb{R}$, and $f, g : J \to \mathbb{R}$ Riemann Integrable functions. Show that $cf + dg$ is Riemann Integrable on J.

2. Let J be a rectangle in $\mathbb{R}^n$, and $f : J \to \mathbb{R}$ a bounded fuction. Show that f is Riemann Integrable on J **if and only if** for every $\epsilon > 0$, there exists a finite non-overlapping exact cover $\mathcal{C}$ of $[a, b]$ such that
$$\mathcal{U}(f; \mathcal{C}) - \mathcal{L}(f; \mathcal{C}) < \epsilon.$$

3. Suppose $f : [a, b] \to \mathbb{R}$ is a bounded monotone function. Show that f is Riemann Integrable.

4. Let $f : [a, b] \to \mathbb{R}$ be a bounded function, and suppose that f is continuous except at the points $t_1 < t_2 < \cdots < t_n$. Show that f is Riemann Integrable.