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# Measure and Integration Exercises 4

1. Let $A, B \subseteq \mathbb{R}^N$, and suppose that $A \subseteq B$ and $|B \setminus A|_e = 0$. Show that if $A$ is measurable, then $B$ is measurable and $|A| = |B|$.

2. Prove that $|x + E|_e = |E|_e$ for all $x \in \mathbb{R}^N$ and every $E \subseteq \mathbb{R}^N$.

3. Let $A \subseteq \mathbb{R}^M$. The *inner Lebesgue measure* of $A$ is defined by

$$|A|_i = \sup\{|K|_e : K \subseteq A, K \text{ is compact }\}.$$

Prove the following.

(a) $|A|_i \leq |A|_e$ for all $A \in \mathbb{R}^M$.
(b) If $A \subseteq B$, then $|A|_i \leq |B|_i$.
(c) If $A_1, A_2, \ldots$ are disjoint, then $|\bigcup_{n=1}^{\infty} A_n|_i \geq \sum_{n=1}^{\infty} |A_n|_i$.
(d) If $A$ is compact or open, then $|A|_e = |A|_i$.