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Measure and Integration Exercises 5

1. Suppose $A_1, A_2 \subseteq \mathbb{R}^N$ are Lebesgue measurable.
   (a) Show that if $A_1 \subseteq A_2$ and $|A_1| < \infty$, then $|A_2 \setminus A_1| = |A_2| - |A_1|$.
   (b) Show that if $|A_1 \cap A_2| < \infty$, then $|A_1 \cup A_2| = |A_1| + |A_2| - |A_1 \cap A_2|$.
2. Let $\{\Gamma_n\}_{n=1}^{\infty}$ be a sequence of Lebesgue measurable subsets of $\mathbb{R}^N$.
   (a) Show that if $|\Gamma_n \cap \Gamma_m| = 0$ for $n \neq m$, then $|\bigcup_{n=1}^{\infty} \Gamma_n| = \sum_{n=1}^{\infty} |\Gamma_n|$.
   (b) Show that if $\Gamma_1 \subseteq \Gamma_2 \subseteq \ldots$, then $|\bigcup_{n=1}^{\infty} \Gamma_n| = \lim_{n\to\infty} |\Gamma_n|$.
   (c) Show that if $|\Gamma_1| < \infty$ and $\Gamma_1 \supseteq \Gamma_2 \supseteq \ldots$, then $|\bigcap_{n=1}^{\infty} \Gamma_n| = \lim_{n\to\infty} |\Gamma_n|$.
3. Let $A \subseteq \mathbb{R}^N$ be Lebesgue measurable. Show that there exists a sequence $K_1 \subseteq K_2 \subseteq K_3 \subseteq \ldots$ of compact subsets of A such that $|A \setminus \bigcup_{n=1}^{\infty} K_n| = 0$.