d) What is the probability that the majority of the sample will be pro 134? (Majority means strictly more than half.)

8. Cards are dealt from a well-shuffled standard deck until the first heart appears.

   a) What is the probability that exactly 5 deals are required?

   b) What is the probability that 5 or fewer deals are required?

   c) What is the probability that exactly 3 deals were required, given that 5 or fewer were required?

9. Suppose events A, B, and C are independent with probabilities 1/5, 1/4, and 1/3, respectively. Write down numerical expressions for the following probabilities:

   a) $P(A$ and B and $C)$

   b) $P(A$ or B or $C)$

   c) $P($exactly one of the three events occurs$)$

10. The four major blood types are present in approximately the following proportions in the population of the U.S.A.

Type	A	B	AB	0
proportion	42%	10%	4%	44%

Note that each person's blood is exactly one of these four types. Type AB is a separate type, not the intersection of type A and type B.

   a) If two people are picked at random from this population, what is the chance that their blood is of the same type? Of different types?

   b) If four people are picked at random, let $P(k)$ be the chance that there are exactly k different blood types among them. Find $P(k)$ for $k = 1, 2, 3, 4$.

11. A hat contains n coins, f of which are fair, and b of which are biased to land heads with probability 2/3. A coin is drawn from the hat and tossed twice. The first time it lands heads, and the second time it lands tails. Given this information, what is the probability that it is a fair coin?

12. Suppose n ordinary dice are rolled.

   a) What is the chance that the dice show n different faces?

   b) What is the chance that at least one number appears more than once?

13. **Formula for $P(A|B)$ by conditioning on cases of B.** Show if $B_1, \ldots, B_n$ is a partition of B, then

$$P(A|B) = P(A|B_1)P(B_1|B) + \cdots + P(A|B_n)P(B_n|B)$$

14. There are 100 boxes, and for each $i = 1, 2, \ldots, 100$, box i contains proportion $i/100$ of gold coins (the rest are silver). One box is chosen at random, then a coin is drawn at random from this box.

   a) If the coin drawn is gold, which box would you guess was chosen? Why?