10 ~~[illegible]~~

~~[illegible]~~

We weten dat $A^t = -A$, dus laat v eigenvector zijn met λ eigenwaarde

$v^t A^t v = v^t A^t v \Rightarrow$

$(Av)^t v = -v^t A v \Rightarrow$

$(\lambda v)^t v = -v^t \lambda v \Rightarrow$

$\lambda v^t \cdot v = -\lambda v^t v \Rightarrow$

$\lambda \langle v^t, v\rangle = -\lambda \langle v^t, v\rangle \Rightarrow$ $v \neq 0$, dus $\langle v^t, v\rangle \neq 0$

$\lambda = -\lambda \Rightarrow \lambda = 0$ □

6 $v_1 = 1,\ v_2 = x,\ v_3 = x^2$

$u_1 = v_1 = 1$

$u_2 = v_2 - \frac{\langle u_1, v_2\rangle}{\langle u_1, u_1\rangle} \cdot u_1$

$= x - \frac{\int_0^1 x\,dx}{\int_0^1 1\cdot dx} \cdot 1 = x - \frac{1}{2} \Rightarrow \varepsilon_2 = \frac{u_2}{\|u_2\|}$

$u_3 = v_3 - \frac{\langle u_1, v_3\rangle}{\langle u_1, u_1\rangle} \cdot u_1 - \frac{\langle u_2, v_3\rangle}{\langle u_2, u_2\rangle} \cdot u_2$

$= x^2 - \frac{\int_0^1 x^2\,dx}{1} \cdot 1 - \frac{\int_0^1 x^3 - \frac{x^2}{2}\,dx}{\int_0^1 (x-\frac{1}{2})^2\,dx} \cdot (x - \frac{1}{2})$

$= x^2 - \frac{1}{3} - \frac{\frac{1}{12}}{\frac{1}{12}}(x - \frac{1}{2})$

$= x^2 - x + \frac{1}{6} \Rightarrow \varepsilon_3 = \frac{u_3}{\|u_3\|}$

Ter controle:

$\langle u_1, u_2\rangle = \int_0^1 x - \frac{1}{2}\,dx = \frac{1}{2} - \frac{1}{2} = 0$

$\langle u_1, u_3\rangle = \int_0^1 x^2 - x + \frac{1}{6}\,dx = \frac{1}{3} - \frac{1}{2} + \frac{1}{6} = 0$

$\langle u_2, u_3\rangle = \int_0^1 (x - \frac{1}{2})(x^2 - x + \frac{1}{6})\,dx$

$= \int_0^1 x^3 - \frac{3}{2}x^2 + \frac{4}{6}x - \frac{1}{12}\,dx$

$= \frac{1}{4} - \frac{3}{6} + \frac{4}{12} - \frac{1}{12} = 0$

$(\varepsilon_1, \varepsilon_2, \varepsilon_3) = (1, \sqrt{3}(2x-1), 6\sqrt{5}(x^2 - x + \frac{1}{6}))$

17 Laat $u = (0,0,0,1)$, dan

$\langle u, u \rangle = -1 < 0$ ↯

inproduct mag niet negatief zijn

Dit definieerd dus ~~geen~~ inproduct.

b) $v_1 = (1,0,1,0)$, $v_2 = (1,0,0,1)$

$u_1 = v_1 \qquad \varepsilon_1 = \frac{v_1}{\|v_1\|} = \frac{1}{\sqrt{4}} (1,0,1,0) = \left(\frac{1}{2}, 0, \frac{1}{2}, 0\right)$

$$u_2 = v_2 - \frac{\langle u_1, v_2 \rangle}{\langle u_1, u_1 \rangle} \cdot u_1$$

$$= (1,0,0,1) - \frac{1}{4} \cdot (1,0,1,0)$$

$$= \left(\frac{3}{4}, 0, -\frac{1}{4}, 1\right)$$

$$\varepsilon_2 = \frac{u_2}{\|u_2\|} = \frac{\frac{1}{4}\cdot(3,0,-1,4)}{\sqrt{\frac{9}{16}+\frac{3}{16}+4}} = \left(\frac{3}{2\sqrt{19}}, 0, \frac{-1}{2\sqrt{19}}, \frac{2}{\sqrt{19}}\right)$$

Met Gram-Schmidt.

10a) $T_\varepsilon^\varepsilon = \begin{pmatrix} 2 & -2 \\ -2 & 5 \end{pmatrix}$

$$\begin{vmatrix} 2-\lambda & -2 \\ -2 & 5-\lambda \end{vmatrix} = (\lambda_1 - 6)(\lambda_2 - 1) = 0$$

Als $\lambda_1 = 6$

$$\begin{pmatrix} -4 & -2 \\ -2 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow v_1 = (-1, 2)$$

$$\|v_1\| = \sqrt{1^2+2^2} = \sqrt{5}$$

$a_1 = \left(\frac{-1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right)$

Als $\lambda_2 = 1$

$$\begin{pmatrix} 1 & -2 \\ -2 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow v_2 = (2, 1)$$

$$\|v_2\| = \sqrt{5}$$

$a_2 = \left(\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right)$

Kies nu P met a_1 en a_2 als kolommen

$P = \begin{pmatrix} -1 & 2 \\ 2 & 1 \end{pmatrix}$, dan geldt:

$$T_\alpha^\alpha = P^{-1} \cdot T_\varepsilon^\varepsilon \cdot P = \begin{pmatrix} 6 & 0 \\ 0 & 1 \end{pmatrix}$$

18 $\mathbb{R}^n$ is de gehele vectorruimte, dus

als $W \subset U$, dan $U^{\perp \mathbb{R}^n} \subset W^{\perp \mathbb{R}^n}$

$\subset$

Als $x \in W$, dan $x \notin W^{\perp U}$, dus $x \in (W^{\perp U})^{\perp \mathbb{R}^n}$

Als $x \in U^{\perp \mathbb{R}^n}$, ~~dan~~ Er geldt $W^{\perp U} \subset U$,

dus $U^{\perp \mathbb{R}^n} \subset (W^{\perp U})^{\perp \mathbb{R}^n}$, dus dan zit $x \in (W^{\perp U})^{\perp \mathbb{R}^n}$

Dus $U^{\perp \mathbb{R}^n} + W \subset (W^{\perp U})^{\perp \mathbb{R}^n}$

Nu met dimensieanalyse:

Laat $\dim(U) = m$, $\dim(W) = m'$

Met $1 \leq m' \leq m \leq n$

$\dim(U^{\perp \mathbb{R}^n}) = n - m$, $\dim(W) = m'$

dus omdat $U^{\perp \mathbb{R}^n} \cap W = \emptyset$ geldt

$\dim(U^{\perp \mathbb{R}^n} + W) = n - m + m'$

$\dim(W^{\perp U}) = m - m'$, dus $\dim((W^{\perp U})^{\perp \mathbb{R}^n}) = n - (m - m')$

Dus ze hebben dezelfde dimensie en omdat

$U^{\perp \mathbb{R}^n} + W \subset (W^{\perp U})^{\perp \mathbb{R}^n}$ geldt dat

$U^{\perp \mathbb{R}^n} + W = (W^{\perp U})^{\perp \mathbb{R}^n}$

17 Laat $u = (0,0,0,1)$, dan

$\langle u, u \rangle = -1 < 0$ ↯

inproduct mag niet negatief zijn.

Dit definieerd dus geen inproduct.

b) $v_1 = (1,0,1,0)$, $v_2 = (1,0,0,1)$

$u_1 = v_1$ $\quad \varepsilon_1 = \frac{v_1}{\|v_1\|} = \frac{1}{\sqrt{4}}(1,0,1,0) = (\frac{1}{2}, 0, \frac{1}{2}, 0)$

$u_2 = v_2 - \frac{\langle u_1, v_2 \rangle}{\langle u_1, u_1 \rangle} \cdot u_1$

$= (1,0,0,1) - \frac{1}{4} \cdot (1,0,1,0)$

$= (\frac{3}{4}, 0, -\frac{1}{4}, 1)$

$\varepsilon_2 = \frac{u_2}{\|u_2\|} = \frac{\frac{1}{4} \cdot (3,0,-1,4)}{\sqrt{\frac{9}{16} + \frac{3}{16} + 4}} = (\frac{3}{2\sqrt{19}}, 0, \frac{-1}{2\sqrt{19}}, \frac{2}{\sqrt{19}})$

Met Gram-Schmidt.