Seminar on Logic. Exercise to be handed in 8th of May

1. (3pt.) Show that the subobject R defined in the proof of Proposition 7.4.3. is actually an equivalence relation.

2. (2pt.) Show that filling in $P = \Gamma(X, \mathcal{O}_X)$ and $Q = \Gamma(Y, \mathcal{O}_Y)$ in (iii) gives us (iv) in the proof of 7.4.5.

3. (4.5pt.) In the proof of Proposition 7.2.5. we directly used a characterization of what it means for $\mathscr{F}$ to be a sheaf. Prove that this is a valid characterization, i.e., show the following:
   (a) A compatible family in $\mathscr{F}$ at $(X, \mathcal{O}_X)$ is a collection of elements $s_i \in \mathscr{F}(X_i, \mathcal{O}_{X_i})$ satisfying the condition given by $(*)$.
   (b) $\mathscr{F}$ being separated as a presheaf is equivalent to the induced map $\mathscr{F}(X, \mathcal{O}_X) \to \prod_{i \in I} \mathscr{F}(X_i, \mathcal{O}_{X_i})$ being injective for a covering $\{(X_i, \mathcal{O}_{X_i}) \to (X, \mathcal{O}_X)\}_{i \in I}$.
   (c) $\mathscr{F}$ being a sheaf is then equivalent to the existence of a unique element $s \in \mathscr{F}(X, \mathcal{O}_X)$ with image s_i.

4. (0.5pt.) In sections 7.4 and 7.5 there are numerous typos to be found. Spot one of them, explain why it is wrong and fix it.