Seminar on Models of Intuitionism

Hand-in exercise 11

11 May (due 18 May)

Exercise 1. Let G be some fixed group with neutral element e. We say that G *has a (left) action on a set* X if there is some operation $G \times X \to X$ denoted by $(g, x) \mapsto g \cdot x$ such that for any $x \in X$ and $g, h \in G$ the following equalities hold: $e \cdot x = x$ and $h \cdot (g \cdot x) = (hg) \cdot x$. If G acts on a set X, then we call X a *G-set*.

If X and Y are two G-sets, then an *equivariant map* (or *G-map*) is a map of sets $f \colon X \to Y$ such that $g \cdot f(x) = f(g \cdot x)$ for any $x \in X$ and $g \in G$.

In this exercise we will consider the category G-Set whose objects are G-sets and whose morphisms are G-maps (with function composition) [1]. One can show that this category is connectionally closed. The exercises ask you to partly verify this.

A general hint for the exercises: the forgetful functor $U\colon$ G-$\mathsf{Set} \to \mathsf{Set}$ which sends each G-set to its underlying set is a c.c. functor.

(a) Show that the category G-Set has products. *(2 points)*

(b) Prove that G-Set has exponentials. (Hint: Start by examining the evaluation arrow.) *(3 points)*

Exercise 2. For the equations (4), (5), (7), (10) and (12) from the handout, write down the two deductions that the equation identifies. *(5×1 point)*

[1] Fun fact for category theory lovers: this category is equivalent to the functor category Set^G (where we view G as a category with a single object and group elements as arrows). The requirement on G-maps is simply the naturality of the natural transformations. If one were to consider right group actions, then this category is equivalent to the category of presheaves on G.