

Seminar on models of Intuitionism

Hand-in exercise 7

March 30, 2017

Exercise 1 Let φ be the formula

$$((\neg\neg P \rightarrow P) \rightarrow (\neg P \vee \neg\neg P)) \rightarrow (\neg P \vee \neg\neg P)$$

- (a) (*4 points*) Show that φ is identically solvable.
- (b) (*3 points*) Show that φ is not derivable in Intuitionistic propositional logic, and conclude that the Medvedev model of finite problems as proposed in the lecture is not complete.

Exercise 2 (*3 points*) Show that every critical implication is refutable.

Bonus exercise (*0 points*) Prove that for every formula A , we have $\vdash A$ or $A \vdash J$ with J a critical implication. You may restrict to the case where A is of the form $B \rightarrow C$ and B and C are both not derivable.

As you can see this exercise is not worth any points. I would however be more than happy to evaluate any attempts that you want to make. A complete and correct solution will be awarded with a Mars candybar.