

Hand-in exercises for theorem 1.5 and section 2

To be handed in on the 18th of April

Exercise 1. In the proof of theorem 1.5 of Devlin, we constructed an isomorphism π with some properties. Prove the following:

1. $\pi(\omega_1) = \alpha$
2. $\pi(\mathbf{T}) = \mathbf{T} \upharpoonright \alpha$
3. $\pi(A) = A \cap (T \upharpoonright \alpha)$

Exercise 2. Show that, at the bottom of page 116 in the proof of theorem 1.5 of Devlin, the sets A_α and b_x , as defined in the definition of T_α , exist for every $\alpha < \omega_1$ and $x \in T \upharpoonright \alpha$.

Exercise 3. Let $\mathbb{S} = \langle S, \leq \rangle$ be a Souslin tree. Define the product $\mathbb{S} \times \mathbb{S}$ as the poset $\langle X, \preceq \rangle$, where $X = \{(s, t) \mid s, t \in S \wedge \text{ht}(s) = \text{ht}(t)\}$ and $(a, b) \preceq (a', b')$ if and only if $a \leq a'$ and $b \leq b'$. Show that $\mathbb{S} \times \mathbb{S}$ is not a Souslin tree.