Seminar on Set Theory

Hand-in exercise 3

October 2, 2015

Exercise 1

Let R be a well-founded relation on V, the universe of sets. A function on V is a class of ordered pairs which defines a single-valued mapping of V into V. The principle of recursion on a well-founded relation R is the assertion that if F is any function on V, then there is a function G on V such that

$$\forall u[G(u) = F(\langle u, G|Ru\rangle)].$$

Prove that the principle of recursion on well-founded relations holds in ZF.

Exercise 2

Let $V^{(B)}$ be the universe of B-valued sets for some fixed complete Boolean algebra B.
By Theorem 1.17(ii), it holds that $u(x) \leq [\![x \in u]\!]$ for all $u \in V^{(B)}$ and $x \in \mathrm{dom}(u)$.

(a) Show that there are elements $u \in V^{(B)}$ and $x \in \mathrm{dom}(u)$ such that $u(x)$ is *strictly* less than $[\![x \in u]\!]$.

The elements $v \in V^{(B)}$, such that $v(y) = [\![y \in v]\!]$ for all $y \in \mathrm{dom}(v)$ are called *extensional*.

(b) Let $u \in V^{(B)}$ and let $v = \{\langle x,\ [\![x \in u]\!]\rangle \mid x \in \mathrm{dom}(u)\} \in V^{(B)}$. Show that v is extensional with $[\![u = v]\!] = 1$.