Seminar H10: exercises week 11

(Presentation: Nils Donselaar)

Due 9 December 2013

Exercise 1

a) Give a proof of Lemma 2, i.e. prove that if F is a field of characteristic $p \geq 3$, then for all $x \in F(t)$ the expression $u = \frac{x^p + t}{x^p - t}$ has only simple zeroes and poles.

b) Using Lemma 2, complete the proof of Lemma 3 discussed during the presentation by proving the right-to-left direction for the case where $s > 0$ and y is not a p-th power of any function $z \in \overline{F}(t)$.
Hint: Show that v cannot be and at the same time has to be a p-th power under these assumptions, thereby showing that this case cannot occur.

Exercise 2

Prove the Proposition used in the proof of Lemma 4: If $z \in F[t]$ has only simple roots and $t \nmid z$, then $\exists s \in \mathbb{N}_{>0} \; z \mid t^{p^s - 1} - 1$. *Hint: consider $F[t]/(z)$ and use the fact that $n \mid m \Rightarrow t^n - 1 \mid t^m - 1$ at some point in the proof.*