Tame Topology and O-minimal Structures, Homework Set for Triangulation Theorem

Tingxiang Zou (tingxiangzou@gmail.com)

In exercise 1 and 3, we fix an o-minimal expansion $(R, <, \mathcal{S})$ of an ordered real closed field $(R, <, 0, 1, +, -, \cdot)$.

1. (**3 points**) Page 133, Exercise 1.

2. (**3 points**) By the Corollary on page 132, we know that if two definable sets have the same dimension and Euler characteristic, then they are definably equivalent, under the assumption that we are working in a fixed o-minimal expansion $(R, <, \mathcal{S})$ of an ordered real closed field $(R, <, 0, 1, +, -, \cdot)$. Show that the ordered real closed field $(R, <, 0, 1, +, -, \cdot)$ in the assumption cannot be replaced by an ordered abelian group $(R, <, 0, +, -)$.

   More precisely, please give an example of an o-minimal expansion $(R, <, \mathcal{S})$ of an ordered abelian group $(R, <, 0, +, -)$ and two definable sets A, B in it, such that $\dim(A) = \dim(B)$, $E(A) = E(B)$, but there is no definable bijection from A onto B. (**Hint:** see the example in (7.11) chapter 1, p.28.)

3. (**4 points**) (A taste of Trivialization Theorem) Page 134, Exercise 4. (You can use the result in Exercise 3 on page 134 freely/without proving it.)