The Logic of Interaction

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Nijmegen, 13/09/10

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Me and my research

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How can we write better software?

Static typing

Model checking

Theorem proving

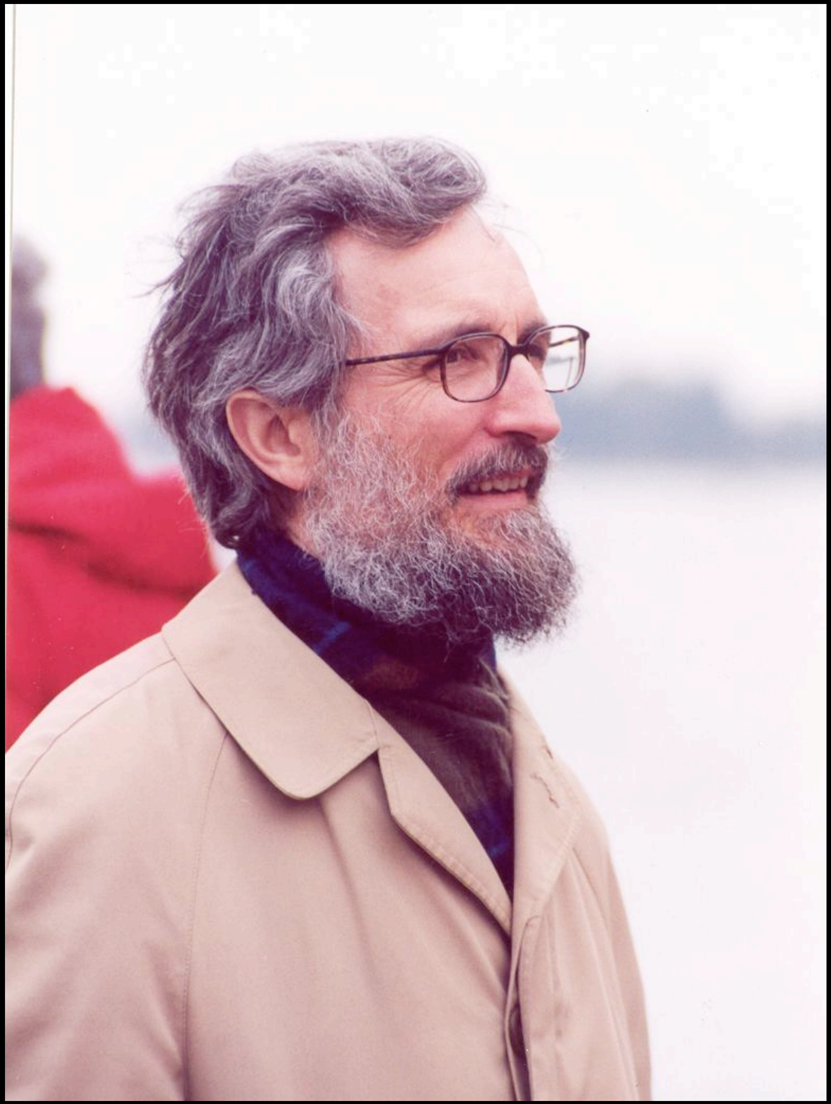
How can we write better software?

Automatic testing

Static analysis

Best software engineering practices

Type Theory
Per Martin-Löf

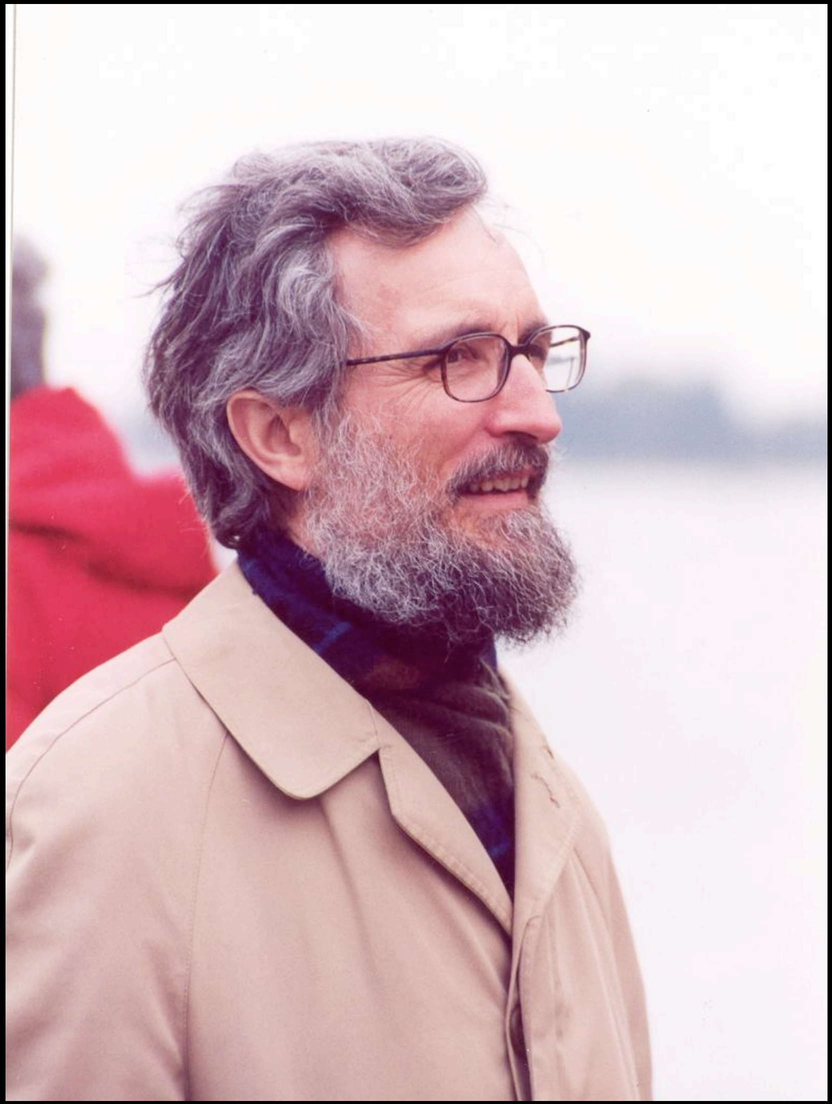


- A foundation of constructive mathematics;
- a functional programming language.

Type Theory
Per Martin-Löf

- A foundation of constructive mathematics;
- a functional programming language.

Really?



What about...

- mutable references?
- arrays?
- exceptions?
- concurrency?
- a GUI?
- a foreign function interface?
- network communication?
- a compiler?
- general recursion?
- file manipulation?
- random numbers?
- ...

PhD Thesis

- **Goal:** Reason about effectful programs.
- **Solution:** Implement a pure and total specification of effects in type theory. Replace specs with “real effects” on compilation.
- **Result:** Write and reason about effectful programs.

What’s missing?

- I’ve studied this approach for individual effects – but what is the *common theme*?
- Proofs in type theory can be hard – what *reasoning principles* can we use to make them easier.

This year: common theme

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- Study how Hancock-Setzer *interaction structures* can be used for effectful, dependently-typed programming.

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- Study how Hancock-Setzer *interaction structures* can be used for effectful, dependently-typed programming.

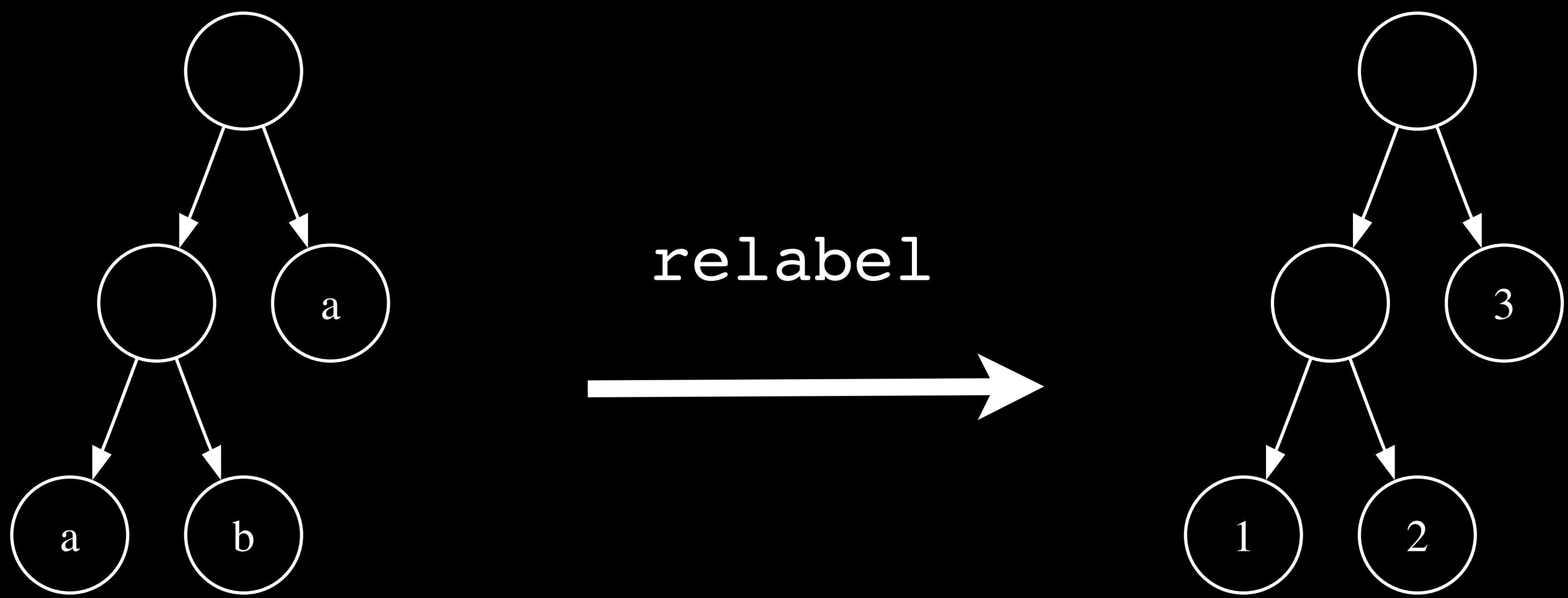
- ... and find grant money to continue this line of research.

Today:
reasoning principle

Relabelling a tree

```
Inductive Tree (a : Set) : Set :=
   | Leaf : a -> Tree a
   | Node : Tree a -> Tree a -> Tree a

relabel : forall a, Tree a -> Tree nat
```


a
a
b
relabel
3
1
2

Relabelling 1.0

```
Fixpoint relabel (a : Set)
  (t : Tree a) (s : nat) : Tree nat * nat
```

Relabelling 1.0

```
Fixpoint relabel (a : Set)
  (t : Tree a) (s : nat) : Tree nat * nat
  := match t with
      | Leaf _ => (Leaf s, s + 1)
      | Node l r =>
         let (l', s') := relabel l s
         in let (r', s'') := relabel r s'
         in (Node l' r', s'')
    end
```

Recursive step

```
| Node l r =>
  let (l', s') := relabel l s
  in let (r', s'') := relabel r s'
  in (Node l' r', s'')
```

Recursive step

```
| Node l r =>
   let (l′, s′) := relabel l s
   in let (r′, s′′) := relabel r s′
   in (Node l′ r′, s′′)
```

Easy to make a mistake!

The state monad

```
(* For some fixed type s *)
Definition State (a : Set) : Type
   := s -> a * s

return : a -> State a
bind : State a
  -> (a -> State b)
  -> State b
```

Return

```
Definition State (a : Set) : Type
   := s -> a * s

Definition return (a : Set)
  : a -> State a :=
      fun x => fun s => (x, s)
```

Bind

```
Definition State (a : Set) : Type
   := s -> a * s

Definition bind (a b : Set)
(c1 : State a) (c2 : a -> State b) : State b
  := fun s => let (x, s') := c1 s
              in c2 x s'
```

Bind

```
Definition State (a : Set) : Type
   := s -> a * s

Definition bind (a b : Set)
(c1 : State a) (c2 : a -> State b) : State b
  := fun s => let (x, s') := c1 s
              in c2 x s'
```

I'll use an infix operator, `>>=` , instead of `bind`

Relabelling 2.0

```
| Node l r =>
   relabel l >>= fun l’ =>
   relabel r >>= fun r’ =>
   return (Node l’ r’)
```

Relabelling 2.0

```
| Node l r =>
   relabel l >>= fun l' =>
   relabel r >>= fun r' =>
   return (Node l' r')
```

No more passing around the state explicitly!

Challenge:
verify the relabelling function,
without expanding the
definitions of return and bind.

Strong specifications

- Strong specifications:
  - define a value;
  - together with a proof that that value satisfies the spec.
- Notation in Coq:

```
{n : nat | n > 7}
```

Program

- Coq's Program framework for working with strong specifications
  - let's you define functions manipulating strongly specified values,
  - and collects assumptions and obligations.
- You need to prove any proof obligations (using tactics) before Program generates a complete Coq term.

Idea:
Decorate the state monad with pre- and postconditions.

Pre- and postconditions

Define the following types:

```
Pre := s -> Prop

Post (a : Set) := s -> a -> s -> Prop
```

The Hoare State Type

Define the following type:

```
HoareState P a Q :=
  {i : s | P i} ->
    {(x,f) : a * s | Q i x f}
```

Remaining questions

- How can we define return?
- How can we define bind?
- How can we use these functions to verify our relabelling function?

Return

```
Definition return (x : a) :
  HoareState
    (fun i => True)
    a
    (fun i y f => i = f /\ x = y)
:= fun i => (x,i)
```

Return

```
Definition return (x : a) :
  HoareState
    (fun i => True)
    a
    (fun i y f => i = f /\ x = y)
:= fun i => (x,i)
```

Need to complete one trivial proof.

Bind - I

```
HoareState P1 A Q1 ->
(A -> HoareState P2 B Q2) ->
HoareState ... B ...
```

What should the pre- and postconditions be?

Bind - II

```
HoareState P1 A Q1 ->
((x:A) -> HoareState (P2 x) B (Q2 x)) ->
HoareState ... B ...
```

What should the pre- and postconditions be?

Bind's precondition

```
\s1 -> P1 s1
    /\ forall x s2, Q1 s1 x s2 -> P2 x s2
```

The initial state must satisfy
the first computation's precondition

Bind's precondition

```
\s1 -> P1 s1
   /\ forall x s2, Q1 s1 x s2 -> P2 x s2
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\s1 -> P1 s1
   /\ forall x s2, Q1 s1 x s2 -> P2 x s2
```

The intermediate state satisfies
the second computation's precondition.

Bind’s precondition

```
\s1 -> P1 s1
   /\ forall x s2, Q1 s1 x s2 -> P2 x s2
```

The intermediate state satisfies
the second computation’s precondition.

Bind's postcondition

```
\s1 y s3 -> exists x, exists s2,
    Q1 s1 x s2 /\ Q2 x s2 y s3
```

There is an intermediate result and an intermediate state relating the two computations.

Implementing bind

- The definition of bind is **exactly the same** as for the state monad...
- ...but we need to fulfill one or two proof obligations.

Using the Hoare State Monad

To verify programs in the state monad, all we need to do is change the type signature, that is, choose the pre- and postconditions.

The program remains unchanged.

Relabelling revisited

```
HoareState
  (fun i => True)
  (Tree nat)
  (fun i t f =>
     flatten t = [i .. i + size t])
```

Relabelling revisited

```
HoareState
  (fun i => True)
  (Tree nat)
  (fun i t f =>
     flatten t = [i .. i + size t]
     /\ f = i + size t)
```

The proof

- The definition gives rise to two proof obligations, one for every case branch.
  - We’ve automated away all work involved in keeping track of the state;
  - The proof for the recursive case is only about 5 lines long (but uses some fancy Program tactics).

Discussion

- Other choices for pre- and postconditions?
- Is the HoareState type a monad?
- Further automation using Ltac?