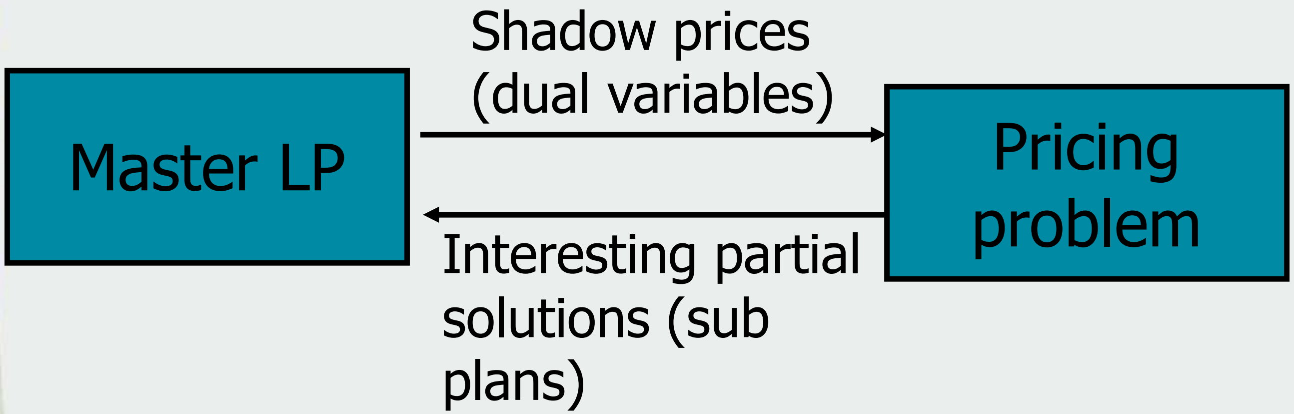
Views on column generation

- Column generation is a method for solving LP's with a large number of variables

- Column generation is (Dantzig-Wolfe) decomposition
  - Master problem: combine partial solutions to a complete solution
  - Pricing (or sub) problem: identify good partial solutions

Column generation: Ping-pong



Dantzig Wolfe decomposition

$$\min cx$$

$$Dx = b_0$$

$$Fx = b_1 \rightarrow P : \{x = \sum_{j \in J} \lambda_j x^j + \sum_{k \in K} \phi_k w^k \mid \sum \lambda_j = 1, \lambda_j, \phi_k \geq 0\}$$

$$x \geq 0$$

You can generalize to multiple polyhedra F_i x = b_i (see book).
This lecture: single polyhedron

Dantzig Wolfe decomposition

$$\min \sum_{j \in J} \lambda_j c x^j + \sum_{k \in K} \phi_k c w^k$$

$$\sum_{j \in J} \lambda_j D x^j + \sum_{k \in K} \phi_k D w^k = b_0$$

$$\sum_{j \in J} \lambda_j = 1$$

$$\lambda_j, \phi_k \geq 0$$

Dual

q (vector)

r

Number of constraints decreased
Number of variables has increased
Column generation!

Pricing problem

reduced cost $\lambda_j : (c - qD)x^j - r \qquad (\forall j \in J)$

reduced cost $\phi_k : (c - qD)w^k \qquad (\forall k \in K)$

recall : columns with negative reduced cost can improve master problem

pricing problem i.e. minimize reduced cost :

$$\min[\min_{j \in J} (c - qD)x^j - r, \min_{k \in K} (c - qD)w^k)] < 0$$

$$\Leftrightarrow$$

$$\min\{(c - qD)x \mid x \in P\} < 0$$

where $P = \{x \mid Fx = b_1 ; x \geq 0\}$,

i.e. 'second' set of constraints in original problem.

Dantzig Wolfe decomposition: Solution algorithm

1. Solve restricted master problem
2. Solve sub (pricing) problem

$$\min(c\text{-}qD)x \quad \text{s.t. } x \in P$$

3. If optimum < r

$$\text{extreme point } x^{j} \text{ with } (c - qD)x^{j} < r, \text{ generate column } \begin{bmatrix} Dx^{j} \\ 1 \end{bmatrix}$$

4. If infinite optimum

$$\text{extreme ray } w^{k} \text{ with } (c - qD)w^{k} < 0, \text{ generate column } \begin{bmatrix} \mathrm{D}\ w^{k} \\ 0 \end{bmatrix}$$

5. If optimum ≥ r do nothing problem solved to optimality, otherwise solve relaxed problem again (go to step 1)

Intermediate lower bound: minimization problem

- z_{opt} optimal LP-value
- z cost of current solution
- In minimization $z \geq z_{opt}$

Column generation might suffer from slow convergence:
tailing -off

Intermediate lower bound helps to cope with tailing-off

- r^* value of dual variable of the convexity constraint
- z_{red} (finite) optimal value of the pricing problem

$$z + (z_{red} - r^*) \leq z_{opt} \leq z$$