Homework for WISB372/ECRMMAT, week 41

Exercise 1 On 12-10 Exercise 3.3 (pp. 143-144) of Bertsekas was started up. Complete it.

Exercise 2 Find the candidate-optimal solutions for the following minimum-time problem: minimize T over all control functions u such that $u(t) \in U := [-1, 1]$, $y(0) = 0$, $\dot{y}(0) = 1$ and $y(T) = 0$ (note: $\dot{y}(T)$ is free). The dynamical system is $\ddot{y} = u$.

Exercise 3 Find the candidate-optimal solutions for the following minimum-time problem: minimize T over all control functions u such that $u(t) \in U := [-1, 1]$, $y(0) = 0$, $\dot{y}(0) = 1$, $y(0) = 0$ and $y(T) = 0$. The dynamical system is $\ddot{y} = u$. Note: this problem is treated in Example 3.4.3 in a general way. You must now find a concrete solution for $y_0 = 0$ and $v_0 = 1$, similar to the other, more concrete minimum-time optimal control problem that was solved in class.

Exercise 4 Find the candidate-optimal solutions for the following minimum-time problem: minimize T over all control functions u such that $\int_0^T u^2(t)dt = 4$, $y(0) = 0$, $\dot{y}(0) = 1$ and $\dot{y}(T) = -1$ (note: $y(T)$ is free and $U = \mathbb{R}$). The dynamical system is $\ddot{y} = u$. *Hint:* The Hamiltionian should be $H = 1 + p_1(t)x_2 + p_2(t)u + \mu u^2$, where $\mu \in \mathbb{R}$ is a Lagrange multiplier. Observe that this is quite similar to what you learned in Dowling, p. 466.

Answer: $T^* = 1$ and $u^* \equiv -2$. This comes from $\mu > 0$ (else MP would not hold) and AE + TV, which give $p_1 \equiv 0$ and $p_2 \equiv c =$ constant. The integral constraint causes $c^2 = 4\mu$, whence $c = \pm 2\sqrt{\mu}$. The case $c = 2\sqrt{\mu}$ leads to $u(t) \equiv -1/\sqrt{\mu}$ and then $y(t) = -\frac{1}{2\sqrt{\mu}}t^2 + \alpha t + \beta$, where $\beta = 0$ and $\alpha = 1$ follow from the conditions for $y(0)$ and $\dot{y}(0)$. Then $\dot{y}(T) = -1$ gives $T/\sqrt{\mu} = 2$. Also, $4 = \int_0^T 1/\mu$ gives $T/\mu = 4$, so it follows that $T = 1$ and $\mu = 1/4$. So $u \equiv= -1/\sqrt{\mu} = -2$. The other case is $c = -2\sqrt{\mu}$. Then a similar reasoning leads to $u \equiv 1/\sqrt{\mu}$ and, inter alia, to $-1 = \dot{y}(T) = T/\sqrt{\mu} + 1$, which is impossible.

Exercise 5 Exercises 3.10 and 3.13 in Bertsekas.

Exercise 6 Find the candidate-optimal solutions for the following discrete-time optimal control problem: maximize $\sum_{k=0}^4 (10x_k - \frac{1}{10}u_k^2)$ over all control vectors (u_0, u_1, u_2, u_4), where the dynamical system is given by $x_{k+1} = x_k + u_k$ and the initial state is $x_0 = 0$. *Hint:* You can apply (3.44) here instead of (3.43), after using the sign trick.