Extra exercises about perfect NE's

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Exercise 1 Prove that if a finite game has precisely one mixed NE (it need not be *completely* mixed), then that NE must be trembling hand perfect.

Exercise 2 Using Exercise **??**, make Exercise 13.13.a in the book in the following formal way.[1] a. In the game of Exercise 13.13, denote the generic mixed strategy of player 1 by $(p, 1-p)$, with $p \in [0,1]$. Likewise, denote by $(q, 1-q)$ and $(r, 1-r)$, with $q, r \in [0,1]$, the mixed strategies of players 2 and 3. With this notation, prove that the expected payoff function $F_i : [0,1]^3 \to \mathbb{R}$ of each player i is given by

$$F_1(p,q,r) := -3pqr + 2pq + 2pr + 2qr + 1 - p - q - r,\ F_2(p,q,r) = q,\ F_3(p,q,r) = r.$$

Derive from these expressions the three best response functions $\beta_1(q,r)$, $\beta_2(p,r)$ and $\beta_3(p,q)$.
b. Conclude from part a that $\{((p, 1-p), (1,0), (1,0)) : 0 \leq p \leq 1\}$ is the set of all mixed NE's.
c. Next, restrict each F_i to $[\frac{1}{t}, 1-\frac{1}{t}]^3$, compute the correspondingly restricted best response functions $\beta_i^{\frac{1}{t}}$, and prove that for each t the unique Nash equilibrium of the perturbed game $G(\frac{1}{t})$ corresponds to $p = q = r = 1 - \frac{1}{t}$.
d. Conclude that the unique trembling hand perfect equilibrium among all mixed strategies is $((1,0),(1,0),(1,0))$.

Exercise 3 Consider the game with bimatrix

$$(A,B) = \begin{pmatrix} (1,0) & (0,2) & (1,3) \\ (0,3) & (0,2) & (0,0) \end{pmatrix}$$

a. Check: of player 2's three pure strategies none is weakly dominated and of player 1's two pure strategies one is weakly dominated.
b. Prove: there is precisely one trembling hand perfect NE for this game.

[1] Alternatively, you can reason similarly to Example 13.23 on p. 183. In fact, that reasoning can be simplified by using Exercise 1: it guarantees the perfectness of the NE (U, L) as soon as the NE (D, R) has been shown to be non-perfect.