Exercise HW5 = 4.4

We consider the "the flea and the comb" example. More precisely, we consider the topological space $X := C \cup \{f\} \subset \mathbb{R}^2$ where $f = (0,1)$ (the flea) and C (the comb) is the union of the sets $[0,1] \times \{0\}$ and $\{\frac{1}{n}\} \times [0,1]$, for $n \in \mathbb{Z}, n \geq 1$. See the picture on page 81 of the lecture notes. The topology of X is induced by the Euclidean topology of R^2.

(a) Show that C is path-connected.

(b) Show that X is connected.

(c) Show that X is not path-connected. Hint: show that any continuous function $\gamma : [0,1] \to X$ with $\gamma(0) = (0,1)$ has to be constant.