

Homework exercise HW2 = 17

Suppose that H is a Lie subgroup of a Lie group G . As usual we denote the Lie algebras of G and H by \mathfrak{g} and \mathfrak{h} , respectively.

(a) If H is normal in G show that

$$\text{Ad}(x)\mathfrak{h} \subset \mathfrak{h}$$

for all $x \in G$. Comment: this exercise is more subtle than it may seem at first. The reason is that H need not be a smooth submanifold of G . Use a suitable characterization of \mathfrak{h} .

(b) If H is normal in G show that \mathfrak{h} is an ideal in \mathfrak{g} .

Now assume that G and H are connected.

(c) Show: if \mathfrak{h} is an ideal in \mathfrak{g} , then H is normal in G .