Mastermath midterm examination
Parallel Algorithms. Retake.

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December 19, 2018

Each of the four questions is worth 10 points. Total time 120 minutes. Motivate your answers!

1. Let $x$ be a vector of length $n \geq 1$.
   (a) [3 pt] Define the cyclic distribution of $x$ over $p$ processors.
   (b) [3 pt] Define the block distribution of $x$ over $p$ processors.
   (c) [2 pt] Give an example of a vector operation where the block distribution is preferred over the cyclic distribution.
   (d) [2 pt] Analyse its BSP cost for both distributions.

2. Let $f$ be a polynomial of degree $n - 1$ in a real variable $x$, which is given by its coefficients $a_0, a_1, \ldots, a_{n-1}$, such that

   \[ f(x) = \sum_{i=0}^{n-1} a_i x^i. \]

   Assume that the coefficients are distributed over the $p$ processors of a parallel computer by the block distribution, where $n \mod p = 0$.
   (a) [5 pt] Design an efficient parallel algorithm for the evaluation of this polynomial for a given value of $x$, where on input $x$ is only available in processor $P(0)$. The output $f(x)$ only needs to become available at $P(0)$. Formulate the algorithm in the notation we have learned, with pseudocode for processor $P(s)$, where $0 \leq s < p$.
   (b) [5 pt] Analyse the BSP cost of your algorithm.
3. Let $A$ be an $n \times n$ dense matrix, distributed by the $q \times q$ square cyclic distribution over $p = q^2$ processors, with $n \mod q = 0$. We want to permute all the rows of the matrix, such that row $i$ moves to the position of row $(i + 1) \mod n$.

(a) [5 pt] Design an algorithm for this permutation of matrix $A$. Formulate the algorithm in the notation we have learned, with pseudocode for processor $P(s, t)$, where $0 \leq s, t < q$.

(b) [5 pt] Analyse the BSP cost of your algorithm.

4. Let $A$ be an $n \times n$ dense matrix, distributed by the $q \times q$ square block distribution $\phi = (\phi_0, \phi_1)$ over $p = q^2$ processors, with $n \mod p = 0$. This means that matrix element $a_{ij}$ is assigned to processor $P(\phi_0(i), \phi_1(j))$. Let $x$ be a vector of length $n$. We want to perform the symmetric rank-1 update $A := A + xx^T$.

(a) [4 pt] Choose a suitable distribution $\phi_x = (\phi_{x,0}, \phi_{x,1})$ for the vector $x$ that will facilitate the desired operation. This means that vector element $x_i$ is assigned to processor $P(\phi_{x,0}(i), \phi_{x,1}(i))$. Note that it must be a true distribution, so every vector component $x_i$ is owned by exactly one processor. Give the reason(s) for your choice.

(b) [3 pt] Design an efficient algorithm for the parallel rank-1 update for the distributions $\phi$ and $\phi_x$. You may describe the algorithm in words, or in pseudocode in the notation we learned.

(c) [3 pt] Analyse the BSP cost of your algorithm.