Principles of Magnetic Resonance Imaging. Matlab session 3

December 2011

Exercise 1: Fold over effect from regular undersampling

Prove the fold-over effect in case of a regular 4-fold undersampling (assume that the total number of sample points, $N$, is a multiple of 4).
Can you do it also for a regular $R$-fold undersampling?

Exercise 2: The Full SENSE matrix. One dimensional encoding

Last week we have constructed the DFT matrix $D$ such that, given the signal $(f_n)$ with $n = 0, \ldots, N - 1$:

$$\hat{f} = Df$$

where $f$ and $\hat{f}$ are the column vectors representing $(f_n)$ and $(\hat{f}_k)$, respectively.

Now we will construct the full SENSE matrix, and study its properties.

- Define a reduction factor, $R$, and the number of coils, $P$ with $P \geq R$.

- Construct the spatially dependent sensitivities function for each of the $P$ coils and store them in a $N \times P$ matrix $C$. Be creative ;-) Start with the case that the sensitivity function is a given constant, that is $S_p = c_p$ for $p = 1, \ldots, P$ and $c_p \in \mathbb{R}$.

- From the DFT matrix $D$, derive the reduced DFT matrix $\tilde{D}$ selecting the rows of $D$ corresponding to the undersampling factor $R$.

- For each coil, construct the diagonal sensitivity matrix $S$, multiply it by $\tilde{D}$ and concatenate vertically the obtained matrices.

The obtained matrix is the full SENSE matrix for one dimensional encoding. Denote it by $D_{\text{full}}$.

For the reconstruction, the matrix $D_{\text{full}}^H D_{\text{full}}$ has to be inverted. The condition number of $D_{\text{full}}$ (matlab function: `cond`) gives a measure of the bad conditioning of the problem. Vary the parameters $R$, $P$ and the coils sensitivity. How does the condition number of $D_{\text{full}}$ vary? Can you explain that? What is the rank of $D_{\text{full}}$? The spectrum of $D_{\text{full}}$ is the set of singular values. Plot the singular values in log scale, what do you see? Use the Matlab function `svd`. Plot the real and the imaginary part of $D_{\text{full}}$ (on a 2D figure, use `imagesc`). How does $D_{\text{full}}$ look like?

Exercise 3: Cartesian SENSE

Download the files `brain.mat`, `SynthCoils.m`, `generate_folded_data.m` and `sense_recon.m` in the attachment of the email you received (if you did not receive it, please let me know and I will send it to you).
Run `generate_folded_data.m` and the cartesian SENSE reconstruction function
`[recon]=sense_recon(imfold,cmap,af);`

Try to understand how the code works:

- What kind of data is stored in `cmap`? How is it generated? What does the parameter `variance` represent?
- What does `imfold` represent? How is it generated?
- How is the unfolding step (least squares problem) performed?
- What do you notice in the unfolded images?

Compute the (relative) error between the reconstructed image and the reference image.
Change the parameters corresponding to

- number of coils
- acceleration (reduction) factor (this should always be a power of 2)
- noise

What do you expect to see? Will the image quality improve or deteriorate? What about the condition numbers of the SENSE matrices? What is the relative error in the reconstructed image?

What happen if you change the coil sensitivities, for instance setting $S_p(\vec{r}) = c_p$ with $p=1,\ldots,P$ and $c_p \in \mathbb{C}$? Or by making them spatially varying only along one dimension, i.e. $S_p(\vec{r}) = f_p(x)$ or $S_p(\vec{r}) = f_p(y)$?
Change also the position of the coils and the width of the sensitivity function. Explain what you see.