

11b. Substitution rule for the N-trivial case

Preparation for Lemma 3.19.

The following substitution routine **subtriv** is more efficient than the use of **subab** followed by putting β equal to zero,

```
In[ * ]:= Clear[subtriv]
subtriv[xx_] := Module[{x}, x = xx //. subtriv0 // Expand;
  x = x //. subtriv0 //. btsub // Simplify]
subtriv0 :=
  {Rna[XX0, ff_] -> 0, Rna[XX1, ff_] -> 0, Rna[XX2, ff_] -> 0, Rna[HHr, ff_] -> t D[ff, t]};
```

```
In[ * ]:= Clear[h, p, r, q]
F = f[t] * Phi[h, p, r, p]
R[Z31, F] // subtriv
R[Z32, F] // subtriv
```

Out[*]= f[t] * Phi[h, p, r, p]

$$\text{Out[*]} = \frac{1}{8 \times (1 + p)} (2 + p + r) \text{Phi}[3 + h, 1 + p, 1 + r, 1 + p] ((h + 2 p - r) f[t] + 2 t f'[t])$$

$$\begin{aligned} \text{Out[*]} = \frac{1}{8 \times (1 + p)} & (- f[t] (2 p (4 - h + 2 p + r) \text{Phi}[3 + h, -1 + p, 1 + r, -1 + p] + \\ & (h + 2 p - r) (2 + p + r) \text{Phi}[3 + h, 1 + p, 1 + r, -1 + p]) + \\ & 2 t (2 p \text{Phi}[3 + h, -1 + p, 1 + r, -1 + p] - (2 + p + r) \text{Phi}[3 + h, 1 + p, 1 + r, -1 + p]) f'[t]) \end{aligned}$$

```
In[ * ]:= sh[3, 1, F, subtriv]
% == sh[3, 1, F, subab]
sh[-3, 1, F, subtriv]
% == sh[-3, 1, F, subab]
sh[3, -1, F, subtriv]
% == sh[3, -1, F, subab]
sh[-3, -1, F, subtriv]
% == sh[-3, -1, F, subab]
```

$$\text{Out[*]} = \frac{1}{8 \times (1 + p)} (2 + p + r) \text{Phi}[3 + h, 1 + p, 1 + r, 1 + p] ((h + 2 p - r) f[t] + 2 t f'[t])$$

```
Out[ * ] = True
```

$$\text{Out[*]} = \frac{1}{8 \times (1 + p)} (2 + p - r) \text{Phi}[-3 + h, 1 + p, -1 + r, 1 + p] ((-h + 2 p + r) f[t] + 2 t f'[t])$$

```
Out[ * ] = True
```

$$\text{Out[*]} = \frac{1}{4 \times (1 + p)} p \text{Phi}[3 + h, -1 + p, 1 + r, -1 + p] ((-4 + h - 2 p - r) f[t] + 2 t f'[t])$$

```
Out[ * ] = True
```

$$\text{Out[*]} = \frac{1}{4 \times (1 + p)} p \text{Phi}[-3 + h, -1 + p, -1 + r, -1 + p] (-((4 + h + 2 p - r) f[t]) + 2 t f'[t])$$

```
Out[ * ] = True
```

Definition of eigenvalues functions on the center of the enveloping algebra

```
In[ * ]:= Clear[ld2, ld3]
ld2[{j_, nu_}] = ld2[j_, nu_] = (nu ^ 2 - 4 + j ^ 2 / 3)
ld3[{j_, nu_}] = ld3[j_, nu_] = (j + 3) (nu ^ 2 - (j - 6) ^ 2 / 9)
```

$$\text{Out[*]} = -4 + \frac{j^2}{3} + nu^2$$

$$\text{Out[*]} = (3 + j) \times \left(-\frac{1}{9} (-6 + j)^2 + nu^2 \right)$$