11e. Eigenfunction equations, N-trivial eigenfunction module

Lemma 3.19 in §3.4

In[ ]:=
```
Clear[sum, p, r, h, f, j, nu]
F = sum[r] × f[r, t] × Phi[h, p, r, p]
```

Out[ ]= f[r, t] × Phi[h, p, r, p] × sum[r]

It turns out that in the N-trivial case the components stay uncoupled.

The computation of the Casimir element takes some time.

In[ ]:=
```
eR[CasZ, F, subtriv] - ld2[j, nu] F // FullSimplify
ft2 = % / (Phi[h, p, r, p] × sum[r]) // Simplify
Coefficient[ft2, f[r, t]] // Factor
Coefficient[ft2, f^(0,1)[r, t]] // Factor
Coefficient[ft2, f^(0,2)[r, t]] // Factor
```

Out[ ]= $\frac{1}{12}$ Phi[h, p, r, p] × sum[r] $\left(\left(-4\left(j^2+3\times\left(-4+nu^2\right)\right)+(h-3\,r)^2\right) f[r,t]+12\,t\left(-3\,f^{(0,1)}[r,t]+t\,f^{(0,2)}[r,t]\right)\right)$

Out[ ]= $\frac{1}{12}\times\left(-4\left(j^2+3\times\left(-4+nu^2\right)\right)+(h-3\,r)^2\right) f[r,t]+t\left(-3\,f^{(0,1)}[r,t]+t\,f^{(0,2)}[r,t]\right)$

Out[ ]= $\frac{1}{12}\times\left(48+h^2-4\,j^2-12\,nu^2-6\,h\,r+9\,r^2\right)$

Out[ ]= $-3\,t$

Out[ ]= t^2

This is the first equation in (3.59)

Weyl-invariance:

In[ ]:=
```
(ft2 /. {j, nu} → S1[{j, nu}]) == ft2 // Simplify
(ft2 /. {j, nu} → S2[{j, nu}]) == ft2 // Simplify
```

Out[ ]= True

Out[ ]= True

The generator in degree 3

In[ ]:=
```
eR[Dt3Z, F, subtriv] - ld3[j, nu] F // FullSimplify
ft3 = % / (Phi[h, p, r, p] × sum[r]) // Simplify
```

Out[ ]= $\frac{1}{72}$ Phi[h, p, r, p] × sum[r] (8 × (3 + j) × (- 6 + j + 3 nu) (j - 3 × (2 + nu)) f[r, t] - (6 + h - 3 r) ((h - 3 r) (h - 3 × (8 + r)) f[r, t] - 36 t (-3 $f^{(0,1)}$[r, t] + t $f^{(0,2)}$[r, t])))

Out[ ]= $\frac{1}{72}$ × (8 × (3 + j) × (- 6 + j + 3 nu) (j - 3 × (2 + nu)) f[r, t] - (6 + h - 3 r) ((h - 3 r) (h - 3 × (8 + r)) f[r, t] - 36 t (-3 $f^{(0,1)}$[r, t] + t $f^{(0,2)}$[r, t])))

It turns out that a simplification can be reached by taking a linear combination with the eigenfunction equation for the Casimir element.

In[ ]:=
```
v = ft3 - (1 / 2) (h - 3 r + 6) ft2 // Simplify
```

Out[ ]= $-\frac{1}{18}$ (h - 2 j - 3 r) ($h^2 + j^2 - 9\, nu^2 + 2 h (j - 3 r) - 6 j r + 9 r^2$) f[r, t]

In[ ]:=
```
Factor[v]
```

Out[ ]= $-\frac{1}{18}$ (h - 2 j - 3 r) (h + j - 3 nu - 3 r) (h + j + 3 nu - 3 r) f[r, t]

In[ ]:=
```
prod = (h - 3 r - 2 j) (h - 3 nu - 3 r + j) (h + 3 nu - 3 r + j)
prod /. {w, nu} → S1[{w, nu}]
prod /. {w, nu} → S2[{w, nu}]
```

Out[ ]= (h - 2 j - 3 r) (h + j - 3 nu - 3 r) (h + j + 3 nu - 3 r)

Out[ ]= (h - 2 j - 3 r) (h + j - 3 nu - 3 r) (h + j + 3 nu - 3 r)

Out[ ]= (h - 2 j - 3 r) (h + j - 3 nu - 3 r) (h + j + 3 nu - 3 r)

Comparison with the routine in Section 11h.

In[ ]:=
```
Clear[h, p, r, f]
efeqt[h, p, r, f] == {ft2, -18 v} // Simplify
```

Out[ ]= True