13b. Generic abelian Fourier term modules

First downward shift operator

In[ ]:=
```
Clear[f, r, t, h, p, r, ff, rr, sol3m1, solm3m1]
F = chbt f[r, t] × Phi[h, p, r, p]
```

Out[ ]= chbt f[r, t] × Phi[h, p, r, p]

In[ ]:=
```
sh[3, -1, F, subab] // compr
sol3m1[rr_] = Solve[% == 0, f[r + 1, t]][[1]] /. r → rr - 1 // Simplify
ff[rr_] = f[rr, t] /. %
```

Out[ ]= $\frac{1}{4 \times (1+p)}$ chbt p Phi[3 + h, −1 + p, r, −1 + p] ((−3 + h − 2 p − r) f[−1 + r, t] + 2 t (2 ⅈ betac π f[1 + r, t] + $f^{(0,1)}$[−1 + r, t]))

Out[ ]= {f[rr, t] → $\frac{1}{4\ \text{betac}\ \pi\ t}$ ⅈ ((−2 + h − 2 p − rr) f[−2 + rr, t] + 2 t $f^{(0,1)}$[−2 + rr, t])}

Out[ ]= $\frac{ⅈ ((-2 + h - 2 p - rr)\ f[-2 + rr, t] + 2\ t\ f^{(0,1)}[-2 + rr, t])}{4\ \text{betac}\ \pi\ t}$

In[ ]:=
```
eimp = efeqa[h, p, -p, f, beta] /. {f[2 - p, t] → ff[2 - p], f^(0,ee_)[2 - p, t] :→ D[ff[2 - p], {t, ee}]} /.
    betac → Conjugate[beta] // Simplify
```

Out[ ]= {$\frac{1}{12}$ × (48 + h^2 − 4 j^2 − 12 nu^2 + 48 p − 6 h p + 21 p^2 − 48 π^2 t^2 Abs[beta]^2) f[−p, t] + t (−((3 + 2 p) $f^{(0,1)}$[−p, t]) + t $f^{(0,2)}$[−p, t]),

(h^3 − 2 j^3 + 18 j nu^2 − 3 h (j^2 + 3 nu^2) + 216 p − $\frac{9 h^2 p}{2}$ − 9 j^2 p − 27 nu^2 p + 216 p^2 + $\frac{135 p^3}{2}$ − 216 p π^2 t^2 Abs[beta]^2) f[−p, t] − 54 p t ((3 + 2 p) $f^{(0,1)}$[−p, t] − t $f^{(0,2)}$[−p, t])}

In[ ]:= Try to simplify

In[ ]:=
```
Coefficient[eimp, f^(0,2)[-p, t]]
```

Out[ ]= {t^2, 54 p t^2}

In[ ]:=
```
eimpb = eimp[[2]] - 54 p eimp[[1]] // Factor
```

Out[ ]= (h − 2 j − 3 p) (h + j − 3 nu − 3 p) (h + j + 3 nu − 3 p) f[−p, t]

Three possible relations

Second downward shift operator

In the same way

```
In[ ]:= Clear[f, r, t, h, p, r, ff, rr]
        F = chbt f[r, t] × Phi[h, p, r, p]
Out[ ]= chbt f[r, t] × Phi[h, p, r, p]
```

```
In[ ]:= sh[-3, -1, F, subab] // compr
        solm3m1[rr_] = Solve[% == 0, f[r - 1, t]][[1]] /. r → rr + 1 // Simplify
        ff[rr_] = f[rr, t] /. %
```

Out[ ]= $-\frac{1}{4\times(1+p)}\text{chbt p Phi}[-3+h,-1+p,r,-1+p]\,(4\,i\,\text{beta}\,\pi\,t\,f[-1+r,t]+(3+h+2p-r)\,f[1+r,t]-2\,t\,f^{(0,1)}[1+r,t])$

Out[ ]= $\left\{f[rr,t]\to\frac{i\,((2+h+2p-rr)\,f[2+rr,t]-2\,t\,f^{(0,1)}[2+rr,t])}{4\,\text{beta}\,\pi\,t}\right\}$

Out[ ]= $\frac{i\,((2+h+2p-rr)\,f[2+rr,t]-2\,t\,f^{(0,1)}[2+rr,t])}{4\,\text{beta}\,\pi\,t}$

In[ ]:= `eip = efeqa[h, p, p, f, beta] /. {f[p - 2, t] → ff[p - 2], f^(0,ee_)[p - 2, t] :> D[ff[p - 2], {t, ee}]} /. betac → Conjugate[beta] // Simplify`

Out[ ]= $\Big\{\frac{1}{12}\times(48+h^2-4j^2-12\,nu^2+48p+6hp+21p^2-48\pi^2t^2\,\text{Abs[beta]}^2)\,f[p,t]+t\,(-((3+2p)\,f^{(0,1)}[p,t])+t\,f^{(0,2)}[p,t]),$
$\left(h^3-2j^3+18\,j\,nu^2-3h\,(j^2+3\,nu^2)-216p+\frac{9h^2p}{2}+9j^2p+27\,nu^2p-216p^2-\frac{135p^3}{2}+216p\,\pi^2t^2\,\text{Abs[beta]}^2\right)f[p,t]+54\,p\,t\,((3+2p)\,f^{(0,1)}[p,t]-t\,f^{(0,2)}[p,t])\Big\}$

In[ ]:= `Coefficient[eip, f^(0,2)[p, t]]`

Out[ ]= $\{t^2,-54\,p\,t^2\}$

In[ ]:= `eip[[2]] + 54 p eip[[1]] // Factor`

Out[ ]= $(h-2j+3p)\,(h+j-3\,nu+3p)\,(h+j+3\,nu+3p)\,f[p,t]$