15c. Check Table 4.1 in Section 4.2.1

Downward shift operators

In[ ]:=
```
Clear[j1, j2]
sh[3, -1, phi[h, p, r, p, nu], subtriv] / phi[h + 3, p - 1, r + 1, p - 1, nu] /. r → (h - 2 j) / 3 //
  Simplify
Solve[% == 0, p]
(h / 3) - p == (2 / 3) j2 /. %[[2]] // Simplify
% /. j2 → -(3 nu + j) / 2
```

Out[ ]=
$$\frac{(h + j + 3\,nu - 3\,p)\,p}{6 \times (1 + p)}$$

Out[ ]=
$$\left\{\{p \to 0\}, \left\{p \to \frac{1}{3}(h + j + 3\,nu)\right\}\right\}$$

Out[ ]=
$$j + 2\,j2 + 3\,nu == 0$$

Out[ ]=
True

In[ ]:=
```
sh[-3, -1, phi[h, p, r, p, nu], subtriv] / phi[h - 3, p - 1, r - 1, p - 1, nu] /. r → (h - 2 j) / 3 //
  Simplify
Solve[% == 0, p]
(h / 3) + p == (2 / 3) j1 /. %[[2]] // Simplify
% /. j1 → (3 nu - j) / 2 // Simplify
```

Out[ ]=
$$-\frac{p\,(h + j - 3\,nu + 3\,p)}{6 \times (1 + p)}$$

Out[ ]=
$$\left\{\{p \to 0\}, \left\{p \to \frac{1}{3}(-h - j + 3\,nu)\right\}\right\}$$

Out[ ]=
$$j + 2\,j1 == 3\,nu$$

Out[ ]=
True

Upward shift operators

```
In[ ]:= q = sh[3, 1, phi[h, p, r, p, nu], subtriv] / Phi[3 + h, 1 + p, 1 + r, 1 + p] // Simplify
        Solve[% == 0, p]
        (* r=-2-p is impossible *)
        h / 3 + p == (2 / 3) j2 - 2 /. %[[2]] /. r → (h - 2 j) / 3 // Simplify
        % /. j2 → -(3 nu + j) / 2
```

$$\text{Out[ ]=}\ \frac{(4 + h + 2\,nu + 2\,p - r) \times (2 + p + r)\,t^{2+nu}}{8 \times (1 + p)}$$

$$\text{Out[ ]=}\ \left\{\{p \to -2 - r\},\ \left\{p \to \frac{1}{2} \times (-4 - h - 2\,nu + r)\right\}\right\}$$

$$\text{Out[ ]=}\ j + 2\,j2 + 3\,nu == 0$$

Out[ ]= True

```
In[ ]:= q = sh[-3, 1, phi[h, p, r, p, nu], subtriv] / Phi[-3 + h, 1 + p, -1 + r, 1 + p] // Simplify
        Solve[% == 0, p]
        h / 3 - p == (2 / 3) j1 + 2 /. %[[1]] /. r → (h - 2 j) / 3 // Simplify
        % /. j1 → (3 nu - j) / 2
```

$$\text{Out[ ]=}\ \frac{(2 + p - r) \times (4 - h + 2\,nu + 2\,p + r)\,t^{2+nu}}{8 \times (1 + p)}$$

$$\text{Out[ ]=}\ \left\{\left\{p \to \frac{1}{2} \times (-4 + h - 2\,nu - r)\right\},\ \{p \to -2 + r\}\right\}$$

$$\text{Out[ ]=}\ j + 2\,j1 == 3\,nu$$

Out[ ]= True