17 Intersection of kernels of downward shift operators (principal series)

17a. Weyl group action

17b. Computations for Lemma 4.5

Part ii)

In[ ]:=
```
{jl + jp, (jp - jl) / 3} /. sub2p // Simplify
% /. sub2r // Simplify
```

Out[ ]= $\left\{\frac{1}{2}(\text{jp} - 3\,\text{nup}), \frac{\text{jp} + \text{nup}}{2}\right\}$

Out[ ]= {-jr, nur}

In[ ]:=
```
{jl + jr, (jr - jl) / 3} /. sub2p // Simplify
```

Out[ ]= {-jp, nup}

In[ ]:=
```
{jp + jr, (jr - jp) / 3} /. sub2p // Simplify
% /. sub2l // Simplify
```

Out[ ]= $\left\{\frac{1}{2}(\text{jp} + 3\,\text{nup}), \frac{1}{2}(-\text{jp} + \text{nup})\right\}$

Out[ ]= {-jl, nul}

In[ ]:=
```
nul + nur /. sub2p // Simplify
```

Out[ ]= nup

Part iii)

Relations

In[ ]:=
```
cc = {{(h + p) / 2, nu1}, {(h - p) / 2, nu2}}
cc //. {h → -jr, p → nur, nu1 → nul, nu2 → nup} /. sub2p // Simplify;
{%[[1]] /. sub2l, %[[2]] /. sub2p} // Simplify
cc //. {h → -jp, p → nup, nu1 → nul, nu2 → nur} /. sub2p // Simplify;
{%[[1]] /. sub2l, %[[2]] /. sub2r} // Simplify
cc //. {h → -jl, p → nul, nu1 → nup, nu2 → nur} /. sub2p // Simplify;
{%[[1]] /. sub2p, %[[2]] /. sub2r} // Simplify
```

Out[ ]= $\left\{\left\{\frac{h+p}{2}, 1+p\right\}, \left\{\frac{h-p}{2}, \frac{1}{2}\times(2-h-p)\right\}\right\}$

Out[ ]= {{-nul, nul}, {-nup, nup}}

Out[ ]= {{nul, nul}, {-nur, nur}}

Out[ ]= {{nup, nup}, {nur, nur}}

Part iv)

In[ ]:=
```
Clear[zt]
cc = {nu1 + j1, nu1 - j1, nu2 + j2, nu2 - j2} /. {j1 → zt1 nu1 - 2 p, j2 → zt2 nu2 + 2 p}
```

Out[ ]= $\left\{1-p+(1+p)\,zt1,\ 1+3p-(1+p)\,zt1,\ \frac{1}{2}\times(2-h-p)+2p+\frac{1}{2}\times(2-h-p)\,zt2,\ \frac{1}{2}\times(2-h-p)-2p-\frac{1}{2}\times(2-h-p)\,zt2\right\}$

In[ ]:=
```
cc1 = cc /. {zt1 → -1, zt2 → -1, nu1 → nul, nu2 → nup} // Simplify
```

Out[ ]= $\left\{1-nul-p,\ 1+nul+3p,\ nup+2p+nup\,zt2,\ 1-\frac{h}{2}+nup-\frac{5p}{2}\right\}$

In[ ]:=
```
cc2 = cc /. {zt1 → 1, zt2 → -1, nu1 → nul, nu2 → nur} // Simplify
```

Out[ ]= $\left\{1+nul-p,\ 1-nul+3p,\ nur+2p+nur\,zt2,\ 1-\frac{h}{2}+nur-\frac{5p}{2}\right\}$

In[ ]:=
```
cc3 = cc /. {zt1 → 1, zt2 → 1, nu1 → nup, nu2 → nur} // Simplify
```

Out[ ]= $\left\{1+nup-p,\ 1-nup+3p,\ nur+2p+nur\,zt2,\ \frac{1}{2}\times(-2+h+2\,nur-3p)\right\}$

In[ ]:=
```
nup - p ≤ p /. p → nur /. sub2p // Simplify
% /. sub2r
```

Out[ ]= $jp \geq 0$

Out[ ]= $\frac{1}{2}(-jr+3\,nur) \geq 0$

In[ ]:=
```
nur - p ≤ p /. p → nup /. sub2p // Simplify
```

Out[ ]= $jp \leq 3\,nup$

In[ ]:=
```
nul - p ≤ p /. p → nup /. sub2p // Simplify
% /. sub2l // Simplify
```

Out[ ]= $jp + 3\,nup \geq 0$

Out[ ]= $jl \leq 0$

In[ ]:=
```
{-p, nul - p} /. p → (h - 2 jl) / 2 // Simplify
{%[[1]] /. h → jl + jp, %[[2]] /. h → jl + jr} // Simplify
% //. sub2p // Simplify
% /. sub2l // Simplify
```

Out[ ]= $\left\{-\frac{h}{2} + jl,\ -\frac{h}{2} + jl + nul\right\}$

Out[ ]= $\left\{\frac{jl - jp}{2},\ \frac{1}{2}(jl - jr + 2\,nul)\right\}$

Out[ ]= $\left\{-\frac{3}{4}(jp + nup),\ -\frac{jp}{2} - nup\right\}$

Out[ ]= $\left\{\frac{3\,(jl + nul)}{4},\ \frac{1}{4} \times (3\,jl + nul)\right\}$

17c. Logarithmic elements