17 Intersection of kernels of downward shift operators (principal series)

17a. Weyl group action

17b. Computations for Lemma 4.5

17c. Logarithmic elements

In the proof of Proposition 4.4 we have to check that logarithmic elements do not contribute to the intersection of the kernels of the downward shift operators.

In[ ]:=
```
Clear[c, h, p, r, j]
ph = (Log[t] + c) Phi[h, p, r, p]
{sh[3, -1, ph, subtriv], sh[-3, -1, ph, subtriv]} // Simplify
eqs = {% /. Log[t] → 0, Coefficient[%, Log[t]]} 4 (p + 1) p^(-1) /.
    Phi[hh_, pp_, rr_, qq_] :→ 1 // Flatten
```

Out[ ]= $(c + \mathrm{Log}[t])\,\mathrm{Phi}[h, p, r, p]$

Out[ ]= $\left\{-\frac{1}{4\times(1+p)} p\,(-2 + c\,(4 - h + 2p + r) + (4 - h + 2p + r)\,\mathrm{Log}[t])\,\mathrm{Phi}[3 + h, -1 + p, 1 + r, -1 + p],\right.$
$\left.-\frac{1}{4\times(1+p)} p\,(-2 + c\,(4 + h + 2p - r) + (4 + h + 2p - r)\,\mathrm{Log}[t])\,\mathrm{Phi}[-3 + h, -1 + p, -1 + r, -1 + p]\right\}$

Out[ ]= $\{2 - c\,(4 - h + 2p + r),\ 2 - c\,(4 + h + 2p - r),\ -4 + h - 2p - r,\ -4 - h - 2p + r\}$

This can be zero only if c≠0

In[ ]:=
```
eqs[[1]] - eqs[[2]] // Simplify
```

Out[ ]= $2\,c\,(h - r)$

This implies r= h

In[ ]:=
```
eqs /. r → h
```

Out[ ]= $\{2 - c\,(4 + 2p),\ 2 - c\,(4 + 2p),\ -4 - 2p,\ -4 - 2p\}$

Since p≥1 this is impossible.