18d. Dimension considerations

Lemma 4.9, F_-

Let F be as in the proof of Proposition 4.8 have components f_r. Let F_+ and F_- be the images of F under the shift operators, and suppose that F_- is a linear combinations of basis functions.

Write $F_- = c_1\,\omega^{0,p-1}(j_2,\,\nu_2) + c_2\,\mu^{0,p-1}(j_2,\,\nu_2)$

In[◦]:=
```
Clear[b, bc]
F = chbt (f[p - 2, t] × Phi[h, p, p - 2, p] + f[p, t] × Phi[h, p, p, p])
sh[3, -1, F, subab] // Simplify
bc = Coefficient[%, chbt Phi[h + 3, p - 1, p - 1, p - 1]] // Simplify
```

Out[◦]= chbt (f[-2 + p, t] × Phi[h, p, -2 + p, p] + f[p, t] × Phi[h, p, p, p])

Out[◦]= $\frac{1}{4\times(1+p)}$ chbt p (f[-2 + p, t] (4 ⅈ betac π t Phi[3 + h, -1 + p, -3 + p, -1 + p] + (-2 + h - 3 p) Phi[3 + h, -1 + p, -1 + p, -1 + p]) + 2 t Phi[3 + h, -1 + p, -1 + p, -1 + p] (2 ⅈ betac π f[p, t] + $f^{(0,1)}$[-2 + p, t]))

Out[◦]= $\frac{1}{4\times(1+p)}$ p ((-2 + h - 3 p) f[-2 + p, t] + 2 t (2 ⅈ betac π f[p, t] + $f^{(0,1)}$[-2 + p, t]))

bc is the highest component of $(S^3)_{-1}\,F$

In[◦]:=
```
sol = Solve[b == bc, f^(0,1)[-2 + p, t]][[1]] // Simplify
```

Out[◦]= {$f^{(0,1)}$[-2 + p, t] → $\frac{1}{2\,p\,t}$ (p (2 - h + 3 p) f[-2 + p, t] + 4 (b + b p - ⅈ betac p π t f[p, t]))}

Substitute this in the second eigenfunction equation

In[◦]:=
```
Clear[sgn]
efeqa[h, p, p, f, beta][[2]] /. j → (h + 3 p) / 2 /. nu → sgn (h - p) / 2 /. sol // Simplify
% /. beta betac → Abs[beta] ^ 2 /. sgn ^ 2 → 1
```

Out[◦]= $\frac{27}{2}$ × (-16 ⅈ b beta (1 + p) π t + p ($-p^2 + p^2\,sgn^2 + h^2\,(-1 + sgn^2) - 2\,h\,p\,(-1 + sgn^2) - 16$ beta betac $\pi^2\,t^2 + 16\,\pi^2\,t^2$ Abs[beta]^2) f[p, t])

Out[◦]= -216 ⅈ b beta (1 + p) π t

Lemma 4.9, F_+

In[ ]:=
```
Clear[bc, b]
F = chbt (f[-p + 2, t] × Phi[h, p, -p + 2, p] + f[-p, t] × Phi[h, p, -p, p])
Fp = sh[-3, -1, F, subab]
bc = Coefficient[Fp, chbt Phi[h - 3, p - 1, 1 - p, p - 1]] // Simplify
```

Out[ ]= chbt (f[2 - p, t] × Phi[h, p, 2 - p, p] + f[-p, t] × Phi[h, p, -p, p])

Out[ ]= $-\frac{1}{4\times(1+p)}$ chbt p (f[2 - p, t] ((2 + h + 3 p) Phi[-3 + h, -1 + p, 1 - p, -1 + p] + 4 ⅈ beta π t Phi[-3 + h, -1 + p, 3 - p, -1 + p]) + 2 t Phi[-3 + h, -1 + p, 1 - p, -1 + p] (2 ⅈ beta π f[-p, t] - $f^{(0,1)}$[2 - p, t]))

Out[ ]= $-\frac{1}{4\times(1+p)}$ p ((2 + h + 3 p) f[2 - p, t] + 4 ⅈ beta π t f[-p, t] - 2 t $f^{(0,1)}$[2 - p, t])

bc is the lowest component of F_+

In[ ]:=
```
sol = Solve[b == bc, f^(0,1)[2 - p, t]][[1]] // Simplify
```

Out[ ]= {$f^{(0,1)}$[2 - p, t] → $\frac{1}{2\,p\,t}$ (p (2 + h + 3 p) f[2 - p, t] + 4 (b + b p + ⅈ beta p π t f[-p, t]))}

This substitution expresses the derivative of f_{2-p} in terms of f_{-p} and b. We substitute it in the second eigenfunction equation.

In[ ]:=
```
efeqa[h, p, -p, f, beta][[2]] /. j → (h - 3 p) / 2 /. nu → (h + p) / 2 /. sol /.
  Abs[beta]^2 → beta Conjugate[beta] // Simplify
```

Out[ ]= -216 ⅈ b (1 + p) π t Conjugate[beta]

The lowest component b of F_+ vanishes, hence $F_+ = 0$.