21c. Identifications for Proposition 4.16 part ii)

We check Lemma 4.17, which is used for this purpose

Relations from Lemma 4.5, applied when two j-values are given

In[ ]:=
```
subst1 =
  {p → (j2 - j1) / 3, j1 → jl, j2 → jp, nu1 → (jr - jp) / 3, nu2 → (jr - jl) / 3, h → jl + jp};
subst2 = {p → (j2 - j1) / 3, j1 → jl, j2 → jr, nu1 → (jr - jp) / 3,
   nu2 → (jp - jl) / 3, h → jl + jr};
subst3 = {p → (j2 - j1) / 3, j1 → jp, j2 → jr, nu1 → (jr - jl) / 3, nu2 → (jp - jl) / 3, h → jp + jr};
```

Parameter comparison

Part i) of the lemma.

In[ ]:=
```
rel = {kap[h, -p] == -m0[j1] - (eps j1 + 1) / 2, s[h, -p] == zt1 nu1 / 2,
        kap[h, p] == -m0[j2] - (eps j2 + 1) / 2,
        s[h, p] == zt2 nu2 / 2} //. parmsub /. m[h, r_] :→ (eps / 2) (r - r0) /.
    {m0[j1] → -eps (r0 - p) / 2, m0[j2] → -eps (r0 + p) / 2} /. h → j1 + j2 // Simplify
```

Out[ ]=
```
{eps (j1 + 3 p) == eps j2, j1 + j2 + p == 2 nu1 zt1, eps (j1 + 3 p) == eps j2, j1 + j2 == p + 2 nu2 zt2}
```

In[ ]:=
```
rel //. subst1 /. jp → -jl - jr /. {zt1 → -1, zt2 → -1} // Simplify
rel //. subst2 /. jp → -jl - jr /. {zt1 → 1, zt2 → -1} // Simplify
rel //. subst3 /. jp → -jl - jr /. {zt1 → 1, zt2 → 1} // Simplify
```

Out[ ]=
```
{True, True, True, True}
```

Out[ ]=
```
{True, True, True, True}
```

Out[ ]=
```
{True, True, True, True}
```

Parts ii) and iii) of Lemma 4.17

In[ ]:=
```
{t^(m0[j1] + 1) WhittakerV[-p - (j1 + 1) / 2, -(h + p) / 4, 2 Pi ell t^2](*choice of sign of s is
        possible *), t^(p + 1) WhittakerV[kap[h, r0], -s[h, r0], 2 Pi ell t^2]
       (*choice of sign of s is possible *), t^(2 + j1 + 2 p + m0[j1]) E^(Pi ell t^2)} //.
      parmsub /. m[h, r_] ⧴ (1 / 2) (r - r0) /. m0[j1] → -eps (r0 - p) / 2 /. j1 → (h - 3 p) / 2 /.
    eps → 1 //. Whrel //. (ell t^2)^ee_ ⧴ ell^ee t^(2 ee) // Simplify
{%⟦1⟧, %⟦2⟧} / %⟦3⟧ // Simplify
```

Out[ ]=
$$\left\{-e^{\frac{1}{4} i \pi (2+h+p-4 i\, ell\, t^2)}\, ell^{\frac{1}{4}(2+h+p)} (2\pi)^{\frac{1}{4}(2+h+p)}\, t^{2+\frac{h}{2}+p-\frac{r0}{2}},\ -e^{\frac{1}{4} i \pi (2+h-r0-4 i\, ell\, t^2)}\, ell^{\frac{1}{4}(2+h-r0)} (2\pi)^{\frac{1}{4}(2+h-r0)}\, t^{2+\frac{h}{2}+p-\frac{r0}{2}},\ e^{ell\, \pi\, t^2}\, t^{2+\frac{h}{2}+p-\frac{r0}{2}}\right\}$$

Out[ ]=
$$\left\{-e^{\frac{1}{4} i (2+h+p) \pi}\, ell^{\frac{1}{4}(2+h+p)} (2\pi)^{\frac{1}{4}(2+h+p)},\ -e^{\frac{1}{4} i \pi (2+h-r0)}\, ell^{\frac{1}{4}(2+h-r0)} (2\pi)^{\frac{1}{4}(2+h-r0)}\right\}$$

The ratios do not depend on t.

In[ ]:=
```
{t^(m0[j2] + 1) WhittakerV[-p + (j2 - 1) / 2, (h - p) / 4, 2 Pi Abs[ell] t^2],
       t^(p + 1) WhittakerV[kap[h, r0], s[h, r0], 2 Pi Abs[ell] t^2],
       t^(2 - j2 + 2 p + m0[j2]) E^(Pi Abs[ell] t^2)} //. parmsub /.
     m[h, r_] ⧴ (-1 / 2) (r - r0) /. m0[j2] → -eps (r0 + p) / 2 /. j2 → (h + 3 p) / 2 /.
   eps → -1 //. Whrel /. (t^2)^ee_ ⧴ t^(2 ee) // Simplify
{%⟦1⟧, %⟦2⟧} / %⟦3⟧ // Simplify
```

Out[ ]=
$$\left\{-e^{-\frac{1}{4} i \pi (-2+h-p+4 i t^2 \mathrm{Abs}[ell])} (2\pi)^{\frac{1}{4}(2-h+p)}\, t^{2-\frac{h}{2}+p+\frac{r0}{2}}\, \mathrm{Abs}[ell]^{\frac{1}{4}(2-h+p)},\ -e^{-\frac{1}{4} i \pi (-2+h-r0+4 i t^2 \mathrm{Abs}[ell])} (2\pi)^{\frac{1}{4}(2-h+r0)}\, t^{2-\frac{h}{2}+p+\frac{r0}{2}}\, \mathrm{Abs}[ell]^{\frac{1}{4}(2-h+r0)},\ e^{\pi t^2 \mathrm{Abs}[ell]}\, t^{2-\frac{h}{2}+p+\frac{r0}{2}}\right\}$$

Out[ ]=
$$\left\{-e^{-\frac{1}{4} i (-2+h-p) \pi} (2\pi)^{\frac{1}{4}(2-h+p)}\, \mathrm{Abs}[ell]^{\frac{1}{4}(2-h+p)},\ -e^{-\frac{1}{4} i \pi (-2+h-r0)} (2\pi)^{\frac{1}{4}(2-h+r0)}\, \mathrm{Abs}[ell]^{\frac{1}{4}(2-h+r0)}\right\}$$

Ratios independent of t