21d. Computations related to Proposition 4.27

Three combinations (j_1, j_2)

This concerns the convention introduced before the statement of Proposition 4.27. See Figure 4.34.

In[∘]:=

```
h < -p /. h → 2 j1 + 3 p //. subst1 /. sub2l // Simplify
-p ≤ h ≤ p /. h → 2 j1 + 3 p //. subst2 /. sub2p // Simplify
h > p /. h → 2 j1 + 3 p //. subst3 /. sub2r // Simplify
```

Out[∘]= nul > 0

Out[∘]= jp ≤ nup && jp + nup ≥ 0

Out[∘]= nur > 0

This confirms the characterization of the three combinations in Figure 4.34.

Table (4.76)

In[∘]:=

```
p4 = {2 p + j1 - nu1, 2 p + j1 + nu1, 2 p - j2 - nu2, 2 p - j2 + nu2};
p4 //. subst1 /. sub2p // Simplify
% /. sub2l // Simplify
```

Out[∘]= {jp - nup, 0, 0, 2 nup}

Out[∘]= {-2 nul, 0, 0, -jl + nul}

In[∘]:=

```
p4 //. subst2 /. sub2p // Simplify
```

Out[∘]= {0, -jp + nup, 0, jp + nup}

In[∘]:=

```
p4 //. subst3 /. sub2p // Simplify
% /. sub2r // Simplify
```

Out[∘]= {0, 2 nup, -jp - nup, 0}

Out[∘]= {0, jr + nur, -2 nur, 0}

Relations for coefficients of k^M

We show the relations (4.80) and (4.74)

In[ ]:=
```
Clear[coA, coB, kap, s]
coA[r_] = -E^(Pi I kap[r]) Gamma[1 + 2 Abs[s[r]]] Gamma[1/2 + Abs[s[r]] - kap[r]]^(-1)
coB[r_] = -I E^(Pi I (kap[r] - Abs[s[r]])) Gamma[1 + 2 Abs[s[r]]] Gamma[1/2 + Abs[s[r]] + kap[r]]^(-1)
```

Out[ ]= $-\frac{e^{i \pi \,\mathrm{kap[r]}}\,\mathrm{Gamma[1 + 2\,Abs[s[r]]]}}{\mathrm{Gamma}\left[\frac{1}{2} + \mathrm{Abs[s[r]] - kap[r]}\right]}$

Out[ ]= $-\frac{i\, e^{i \pi \,(-\mathrm{Abs[s[r]]} + \mathrm{kap[r]})}\,\mathrm{Gamma[1 + 2\,Abs[s[r]]]}}{\mathrm{Gamma}\left[\frac{1}{2} + \mathrm{Abs[s[r]] + kap[r]}\right]}$

In[ ]:=
```
Clear[coV, coW]
coW[r_] = E^(Pi I m[h, r]/2) Sqrt[Factorial[m[h, r]]]
coV[r_] = E^( Pi I m[h, r])/Sqrt[Factorial[m[h, r]]]
```

Out[ ]= $e^{\frac{1}{2} i \pi \,\mathrm{m[h, r]}}\sqrt{\mathrm{m[h, r]!}}$

Out[ ]= $\frac{e^{i \pi \,\mathrm{m[h, r]}}}{\sqrt{\mathrm{m[h, r]!}}}$

In[ ]:=
```
Clear[bt]
bt[r_] = coV[r] × coW[r]^(-1) × coB[r] × coA[r]^(-1) /. kap[rr_] :> -m[h, rr] - eps s[rr] - 1/2 /.
    I^xx_ :> E^(Pi I xx/2) // Simplify
```

Out[ ]= $\left(i\, e^{\frac{1}{2} i \pi \,(-2\,\mathrm{Abs[s[r]]} + \mathrm{m[h, r]})}\,\mathrm{Gamma[1 + Abs[s[r]] + m[h, r] + eps\ s[r]]}\right) \Big/ \left(\mathrm{m[h, r]!\ Gamma[Abs[s[r]] - m[h, r] - eps\ s[r]]}\right)$

We assume eps $(r_0 - h) > 0$ and eps$(r - r_0) \geq 0$.

Then eps s[r]<0.

In[ ]:=
```
bt[r] /. m[h, rr_] :> (eps/2) (rr - r0) /. {Abs[s[r]] → -eps s[r]} /. s[r] → (h - r)/4 /.
    h → r0 - Abs[h - r0]/eps /. Gamma[xx_] :> Factorial[xx - 1] // Simplify
```

Out[ ]= $\frac{i\, e^{-\frac{1}{4} i \pi \,\mathrm{Abs[h-r0]}}}{\left(-1 + \frac{1}{2}\mathrm{Abs[h - r0]}\right)!}$

This confirms (4.80).

In[ ]:=
```
coV[r]/coA[r] /. Gamma[1 + xx_] :> Factorial[xx] // Simplify
```

Out[ ]= $-\frac{e^{-i \pi \,(\mathrm{kap[r]} - \mathrm{m[h, r]})}\,\mathrm{Gamma}\left[\frac{1}{2} + \mathrm{Abs[s[r]] - kap[r]}\right]}{(2\,\mathrm{Abs[s[r]]})!\ \sqrt{\mathrm{m[h, r]!}}}$

This confirms (4.74).