## 22d. Case $m_0(j_2) = 0$, and $\varepsilon = -1$

In[ ]:=
```
m0[j2] /. mrsub
```
Out[ ]=
```
-eps (p + r0)
```

So $r_0 = -p$

In[ ]:=
```
Clear[F, Fp, Fm, f, fp, t, r0]
F = tht[0] × f[-p, t] × Phi[h, p, -p, p]
```
Out[ ]=
```
f[-p, t] × Phi[h, p, -p, p] × tht[0]
```

In[ ]:=
```
Fp = sh[3, -1, F, subnab] // Simplify
```
Out[ ]=

$$\frac{1}{4\times(1+p)} p\,\text{Phi}[3+h, -1+p, 1-p, -1+p]\times\text{tht}[0]\left(\left(-4+h-p+4\,\text{ell}\,\pi\,t^2\right) f[-p,t] + 2\,t\,f^{(0,1)}[-p,t]\right)$$

So the sole component of $F^{+}$ has an expression in terms of $f_p$ and on the other hand has an explicit expression in a V-Whittaker function.

With **cnz** denoting a non-zero factor, and **r0** the quantity $r_0(h)$:

In[ ]:=
```
eq =
  ((-4 + h - p + 4 ell π t^2) f[-p, t] + 2 t f^(0,1)[-p, t] == cnz t^(0 + 1) WhittakerV[-p + 1 + (j2 - 1)/2,
        nu2/2, 2 Pi Abs[ell] t^2]) /. eps → -1 /. hpsub // Simplify
```
Out[ ]=

$$\left(-4+h-p+4\,\text{ell}\,\pi\,t^2\right) f[-p,t] + 2\,t\,f^{(0,1)}[-p,t] == \text{cnz}\,t\,\text{WhittakerV}\left[\frac{1}{4}\times(2+h-p), \frac{1}{4}\text{Abs}[h-p], 2\,\pi\,t^2\,\text{Abs}[\text{ell}]\right]$$

Find expressions for derivatives of $f_p$

In[ ]:=
```
fmpd = f^(0,1)[-p, t] /. Solve[eq, f^(0,1)[-p, t]][[1]] // Simplify
fmpdd = D[fmpd, t] /. Whrel // Simplify
```
Out[ ]=

$$\frac{1}{2\,t}\left(\left(4-h+p-4\,\text{ell}\,\pi\,t^2\right) f[-p,t] + \text{cnz}\,t\,\text{WhittakerV}\left[\frac{1}{4}\times(2+h-p), \frac{1}{4}\text{Abs}[h-p], 2\,\pi\,t^2\,\text{Abs}[\text{ell}]\right]\right)$$

Out[ ]=

$$\frac{1}{16\,t^2}\Bigg(8\times\left(-4+h-p-4\,\text{ell}\,\pi\,t^2\right) f[-p,t] + t\Bigg(-4\,\text{cnz}\left(2+h-p-4\,\pi\,t^2\,\text{Abs}[\text{ell}]\right)\text{WhittakerV}\left[\frac{1}{4}\times(2+h-p), \frac{1}{4}\text{Abs}[h-p], 2\,\pi\,t^2\,\text{Abs}[\text{ell}]\right] + \text{cnz}\left((4+h-p)^2-\text{Abs}[h-p]^2\right)\text{WhittakerV}\left[\frac{1}{4}\times(6+h-p), \frac{1}{4}\text{Abs}[h-p], 2\,\pi\,t^2\,\text{Abs}[\text{ell}]\right] + 8\times\left(4-h+p-4\,\text{ell}\,\pi\,t^2\right) f^{(0,1)}[-p,t]\Bigg)\Bigg)$$

Insert these relations into the eigenfunction equations

In[ ]:=
```
ei = efeqn[h, p, -p, f, ell, 0, -1] /. {nu → nu2, j → j2} /. f^(0,2)[-p, t] → fmpdd /.
    f^(0,1)[-p, t] → fmpd /. hpsub /. ell → -Abs[ell] /. Abs[h - p]^2 → (h - p)^2 // Simplify
```

Out[ ]=
$$\left\{\frac{1}{2}\,\text{cnz}\;t\left(\left(-2-h+p+4\,\pi\,t^{2}\,\text{Abs[ell]}\right)\text{WhittakerV}\left[\frac{1}{4}\times(2+h-p),\;\frac{1}{4}\,\text{Abs}[h-p],\;2\,\pi\,t^{2}\,\text{Abs[ell]}\right]+(2+h-p)\,\text{WhittakerV}\left[\frac{1}{4}\times(6+h-p),\;\frac{1}{4}\,\text{Abs}[h-p],\;2\,\pi\,t^{2}\,\text{Abs[ell]}\right]\right),\;0\right\}$$

This should be zero.

Insert the asymptotic behavior.

In[ ]:=
```
Clear[tau]
ei /. t → Sqrt[tau] / Sqrt[2 Pi Abs[ell]] /. WhittakerV [kap_, s_, tau_] :→
    -E^(-Pi I kap) tau^(-kap) E^(tau / 2) /. Abs[ell] → ell // Simplify
% E^(-tau / 2) tau^(-1 + (h - p) / 4) // Simplify
Limit[%, tau → Infinity]
```

Out[ ]=
$$\left\{-\frac{1}{2\,\sqrt{\text{ell}}\,\sqrt{2\,\pi}}\,i\,\text{cnz}\,e^{-\frac{1}{4}\,i\,(8+h-p)\,\pi+\frac{\text{tau}}{2}}\,\text{tau}^{\frac{1}{4}\times(-4-h+p)}\left(2+2\,\text{tau}-2\,\text{tau}^{2}+h\,(1+\text{tau})-p\,(1+\text{tau})\right),\;0\right\}$$

Out[ ]=
$$\left\{-\left(\left(i\,\text{cnz}\,e^{-\frac{1}{4}\,i\,(8+h-p)\,\pi}\left(2+2\,\text{tau}-2\,\text{tau}^{2}+h\,(1+\text{tau})-p\,(1+\text{tau})\right)\right)\Big/\left(2\,\sqrt{\text{ell}}\,\sqrt{2\,\pi}\,\text{tau}^{2}\right)\right),\;0\right\}$$

Out[ ]=
$$\left\{\frac{i\,\text{cnz}\,e^{-\frac{1}{4}\,i\,(h-p)\,\pi}}{\sqrt{\text{ell}}\,\sqrt{2\,\pi}},\;0\right\}$$

This shows that **cnz** should be zero.