24b. The separate cases

Three m_0 non-negative and l>0

Figures 4.35 and 4.36

$m_0(j)$ decreasing in j

In[ ]:=
```
m0[jr] ≥ 0 /. m0sub[jp] /. eps → 1 /. sub2p // Simplify
```
Out[ ]=
```
jp + 2 m0[jp] ≥ nup
```

Three m_0 non-negative and l<0

Figures 4.35 and 4.36

$m_0(j)$ increasing in j

In[ ]:=
```
m0[jl] ≥ 0 /. m0sub[jp] /. eps → -1 /. sub2p // Simplify
```
Out[ ]=
```
2 m0[jp] ≥ jp + nup
```

Two m_0 non-negative, l>0

Figures 4.37 and 4.39

```
rel = (* for Figure 4.37 *)
  m0[jr] < 0 ≤ m0[jp] < m0[jl] /. m0sub[jl] /. eps → 1 /. sub2p /. sub2l // Simplify
```

Out[ ]=
```
jl + 2 m0[jl] < nul && jl + nul + 2 m0[jl] ≥ 0 && jl + nul < 0
```

In Figure 4.37 (right)we observe a submodule of type $\mathrm{FI}_+(j_l, -\nu_l)$

In[ ]:=
```
1 ≤ nul < -jl && rel // Simplify
```
Out[ ]=
```
nul ≥ 1 && jl + nul < 0 && jl + 2 m0[jl] < nul && jl + nul + 2 m0[jl] ≥ 0
```

For Figure 4.39

In[ ]:=
```
rel = m0[jr] < 0 ≤ m0[jp] == m0[jl] /. m0sub[jl] /. eps → 1 /. sub2p /. sub2l // Simplify
```
Out[ ]=
```
jl + 2 m0[jl] < nul && jl + nul + 2 m0[jl] ≥ 0 && jl + nul == 0
```

In Figure 39 we observe a submodule that can be type $\mathrm{FI}(j_l, j_l)$ (non-tempered repr.) or $\mathrm{FI}(j_l,0)$ (limit of ahol. discr. ser.) We have
$j_l = j_+$ in this figure.

In[ ]:=
```
(* FI(jl,jl) *)
nul == -jl && jl ≤ -1
rel && % // Simplify
```

Out[ ]=
```
nul == -jl && jl ≤ -1
```

Out[ ]=
```
jl + nul == 0 && jl + 2 m0[jl] < nul && jl + nul + 2 m0[jl] ≥ 0 && jl ≤ -1
```

In[ ]:=
```
(* FI(jl,0) *)
jl ≤ 2 && nul == 0
% && rel // Simplify
```

Out[ ]=
```
jl ≤ 2 && nul == 0
```

Out[ ]=
```
False
```

Two m_0 non-negative, l<0

Figure 4.40

In[ ]:=
```
rel = m0[jl] < 0 ≤ m0[jp] < m0[jr] /. m0sub[jr] /. eps → -1 /. sub2p /. sub2r // Simplify
```

Out[ ]=
```
jr + nur > 2 m0[jr] && jr ≤ nur + 2 m0[jr] && jr > nur
```

We observe a submodule of type $I_+(j_r, \nu_r)$

In[ ]:=
```
1 ≤ nur < jr && rel // Simplify
```

Out[ ]=
```
nur ≥ 1 && jr > nur && jr + nur > 2 m0[jr] && jr ≤ nur + 2 m0[jr]
```

Figure 41

In[ ]:=
```
rel = m0[jl] < 0 ≤ m0[jp] == m0[jr] /. m0sub[jr] /. eps → -1 /. sub2p /. sub2r // Simplify
```

Out[ ]=
```
jr + nur > 2 m0[jr] && jr ≤ nur + 2 m0[jr] && jr == nur
```

Two possibilities

In[ ]:=
```
(* IF(jr,0) *)
nur == 0 && jr ≥ 2 && rel // Simplify
```

Out[ ]=
```
False
```

In[ ]:=
```
(* IF(jr,-jr) *)
nur == jr && jr ≥ 1 && rel // Simplify
% /. nur → jr // Simplify
```

Out[ ]=
```
jr == nur && jr ≥ 1 && jr ≤ nur + 2 m0[jr] && jr + nur > 2 m0[jr]
```

Out[ ]=
```
jr ≥ 1 && m0[jr] ≥ 0 && jr > m0[jr]
```

One m_0 non-negative, l>0

Figure 4.42

In[ ]:= **rel = m0[jr] < m0[jp] < 0 ≤ m0[jl] /. m0sub[jl] /. eps → 1 /. sub2p /. sub2l // Simplify**

Out[ ]= nul > 0 && jl + nul + 2 m0[jl] < 0 && m0[jl] ≥ 0

We observe type FI(j_l, v_l)

In[ ]:= **1 ≤ nul < -jl && rel // Simplify**

Out[ ]= nul ≥ 1 && jl + nul < 0 && jl + nul + 2 m0[jl] < 0 && m0[jl] ≥ 0

Figure 4.44

In[ ]:= **rel = m0[jr] == m0[jp] < 0 ≤ m0[jl] /. m0sub[jl] /. eps → 1 /. sub2p /. sub2l // Simplify**

Out[ ]= nul == 0 && jl + nul + 2 m0[jl] < 0 && m0[jl] ≥ 0

Two possibilities

In[ ]:=
```
(* FI(jₗ,0) *)
jl ≤ -1 && nul == 0
% && rel // FullSimplify
```

Out[ ]= jl ≤ -1 && nul == 0

Out[ ]= nul == 0 && m0[jl] ≥ 0 && jl + 2 m0[jl] < 0 && jl ≤ -1

In[ ]:=
```
(* FI(jₗ,jₗ) *)
jl ≤ -1 && nul == -jl && rel // Simplify
```

Out[ ]= False

One m_0 non-negative, l<0

Figure 4.45

In[ ]:= **rel = m0[jl] < m0[jp] < 0 ≤ m0[jr] /. m0sub[jr] /. eps → -1 /. sub2p /. sub2r // Simplify**

Out[ ]= nur > 0 && jr > nur + 2 m0[jr] && m0[jr] ≥ 0

Type IF (jr, nur)

Figure 4.46

In[ ]:= **rel = m0[jl] == m0[jp] < 0 ≤ m0[jr] /. m0sub[jr] /. eps → -1 /. sub2p /. sub2r // Simplify**

Out[ ]= nur == 0 && jr > nur + 2 m0[jr] && m0[jr] ≥ 0

In[ ]:=
```
(* IF(jᵣ,0) *)
jr ≥ 2 && nur == 0 && rel // FullSimplify
```

Out[ ]= nur == 0 && jr ≥ 2 && m0[jr] ≥ 0 && jr > 2 m0[jr]

In[ ]:=
```
(* IF(jᵣ,-jᵣ) *)
jr ≥ 1 && nur == -jr && rel // Simplify
```

Out[ ]= False