

### 3d. Cartan subalgebras

See §3.1

#### Non-compact Cartan subalgebra

Cartan algebra spanned by  $\mathbf{HHi}$  and  $\mathbf{HHr}$

```
In[ ]:= Lb[HHi, HHr]
```

```
Out[ ]:= nul
```

Check of Table 3.3

```
In[ ]:= {Lb[HHr, XX1 + I XX2], Lb[HHi, XX1 + I XX2]} == {XX1 + I XX2, (-3 I) (XX1 + I XX2)} // Simplify
{Lb[HHr, XX1 - I XX2], Lb[HHi, XX1 - I XX2]} == {XX1 - I XX2, 3 I (XX1 - I XX2)} // Simplify
{Lb[HHr, XX0], Lb[HHi, XX0]}
```

```
Out[ ]:= True
```

```
Out[ ]:= True
```

```
Out[ ]:= {2 XX0, nul}
```

```
In[ ]:=
```

#### Compact Cartan subalgebra

Check of Table 3.2

$In[ * ] :=$  **Z12 // . Liesub0**

**Z23 // . Liesub0 // Expand**

**Z13 // . Liesub0 // Expand**

**Z21 // . Liesub0 // Expand**

**Z32 // . Liesub0 // Expand**

**Z31 // . Liesub0 // Expand**

$Out[ * ] =$  **WW1 -  $i$  WW2**

$$Out[ * ] = -\frac{WW1}{2} - \frac{i WW2}{2} + \frac{XX1}{2} + \frac{i XX2}{2}$$

$$Out[ * ] = \frac{i HHi}{2} + \frac{HHR}{2} - i WW0 + i XX0$$

$Out[ * ] =$  **WW1 +  $i$  WW2**

$$Out[ * ] = -\frac{WW1}{2} + \frac{i WW2}{2} + \frac{XX1}{2} - \frac{i XX2}{2}$$

$$Out[ * ] = -\frac{i HHi}{2} + \frac{HHR}{2} + i WW0 - i XX0$$

$In[ * ] :=$  **lb[CKi , Z12]**

**lb[CKi , Z23]**

**lb[CKi , Z13]**

**lb[CKi , Z21]**

**lb[CKi , Z32]**

**lb[CKi , Z31]**

$Out[ * ] =$  **nu1**

$Out[ * ] =$  **3  $i$  Z23**

$Out[ * ] =$  **3  $i$  Z13**

$Out[ * ] =$  **nu1**

$Out[ * ] =$  **-3  $i$  Z32**

$Out[ * ] =$  **-3  $i$  Z31**

$In[ * ] :=$   $\{b[ww0, Z12]$   
 $\{b[ww0, Z23]$   
 $\{b[ww0, Z13]$   
 $\{b[ww0, Z21]$   
 $\{b[ww0, Z32]$   
 $\{b[ww0, Z31]$

$Out[ * ] =$   $2 i Z12$

$Out[ * ] =$   $-i Z23$

$Out[ * ] =$   $i Z13$

$Out[ * ] =$   $-2 i Z21$

$Out[ * ] =$   $i Z32$

$Out[ * ] =$   $-i Z31$